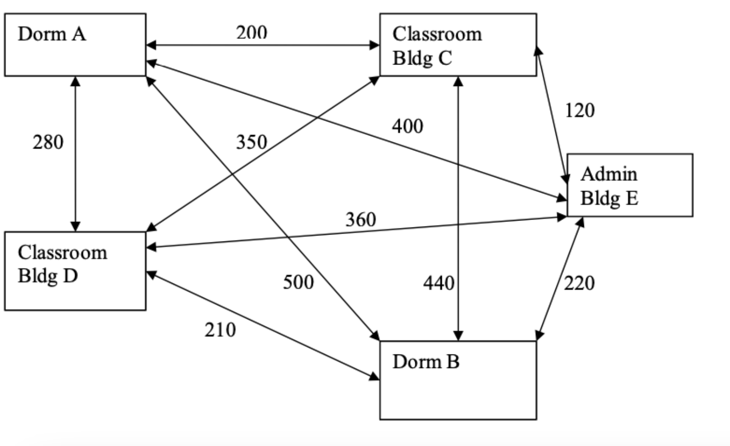
Section 1.4 Networks and Spanning Trees

During the first year on its new campus, Cold Region State College had several unfortunate cases of frostbite among students walking to class. To avoid this happening in the future, the administration intends to convert several of the existing walkways to protected walkways so that students can **go from any building on campus to another building** without being exposed to the inclement weather. To minimize expenses and the time of construction, they wish to draw up plans for the **minimum total length of walkways that need to be converted**. How can this be done? The campus map is shown below with each existing walkway shown. The distances are given in yards.



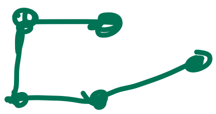
We want a subset of the existing edge so that the graph is still connected, and it includes all vertices and there is exactly one path between any two buildings.

What are we asked to find? A ~~circuit~~? A ~~path~~? Or something else?

minimum spanning tree

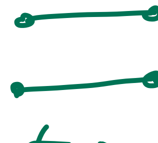
Network - is a connected graph

ex



yes

ex:

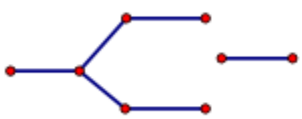


not a network.

Tree - is a network that has no circuits.

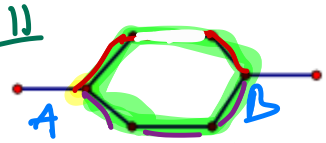
Example 1. Are the following ~~networks~~ graphs a tree? If not, why not?

I



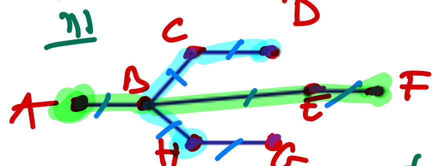
not a tree bc it is not connected (not a network)

II



it is a network but it's not a tree bc it has a circuit.

III



· it is a network

· it has no circuits

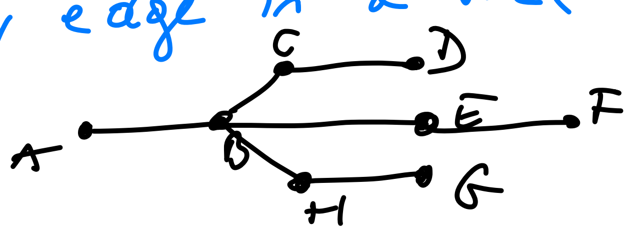
so yes III is a tree.

8V
7 edges

Properties of Trees

1. The Single-Path Property: There is exactly one path between any two vertices in a tree.

2. The All-Bridges Property: Every edge in a tree is a bridge.



3. The N-1 Edges Property:

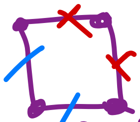
A tree with N vertices has N-1 edges.

ex! III is a tree with 8 vertices, so it has 7 edges.

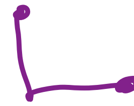
(some of them)

Subtree is a tree that contains a subset of the edges from the original graph

ex



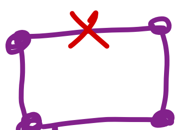
original graph



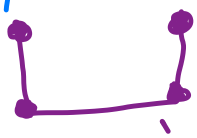
subtree

Spanning Tree is a subtree that contains all of the vertices of the original graph.

ex



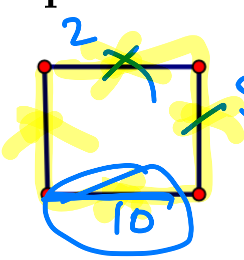
orig. graph



spanning tree

Example 2. Find all possible spanning trees of each graph.

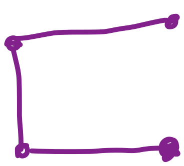
1.



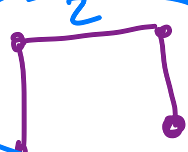
4 spanning trees

I

II

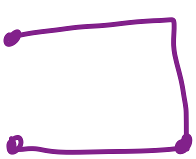


III

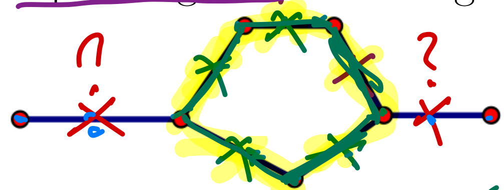


Min. sp. tree.

IV



2.

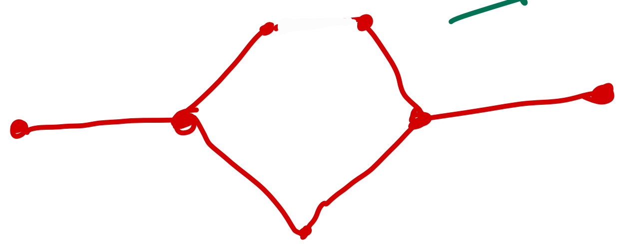


7 verti
7 edges

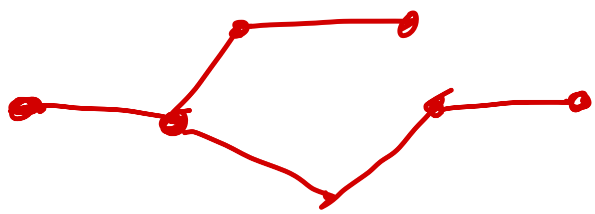
5 spanning trees

7 V
6 edges

I

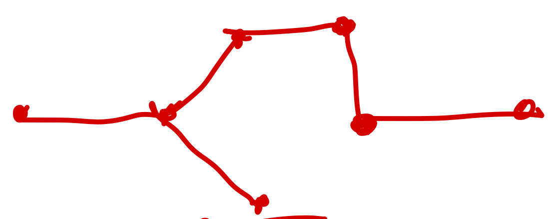


II



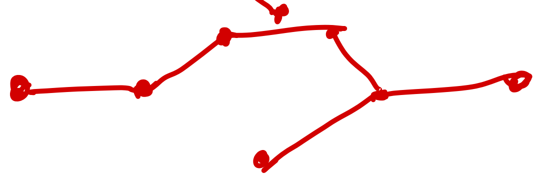
7 V
6 edgs

III



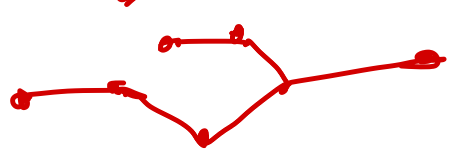
7 V
6 e

IV



7 V
6 e

V

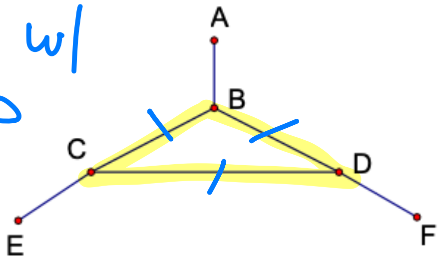


7 V
6 e

Example 3. Find how many spanning trees each graphs has (No need to draw them.)

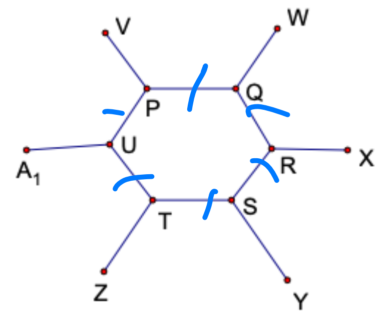
1.

graph has a circuit w/ 3 edges



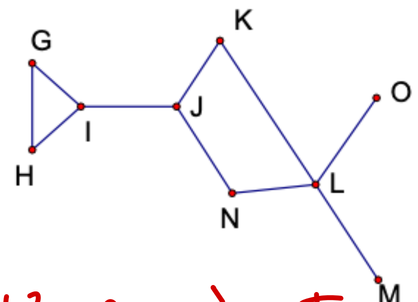
3 spanning trees

2.



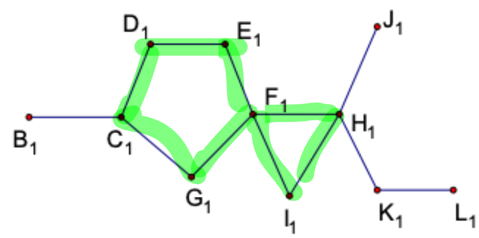
6 spanning trees

3.



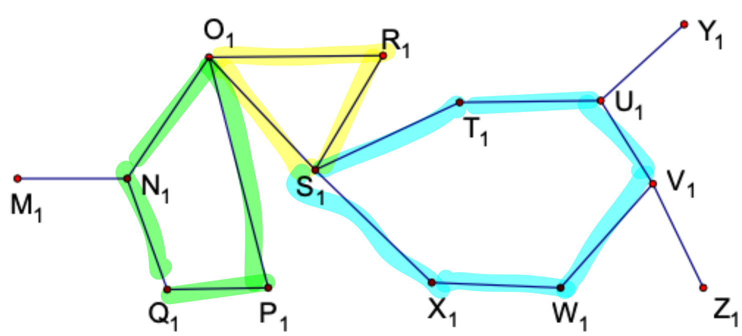
$3 \cdot 4 = 12$ spanning Trees

4.



$5 \cdot 3 = 15$ spanning trees

5.



$4 \cdot 3 \cdot 6 = 72$ spanning Trees.