MTH 122: Practice Exam 1

College of Lake County

Total Possible Points: 83

Name: ______________________________

1. The n^{th} term of a sequence is given. Write the first four terms of the sequence. Express the terms in simplified form when applicable. (4)

   $c_n = 10\left(\frac{-1}{5}\right)^n$

   Solution: $c_1 = 10\left(-\frac{1}{5}\right) = -2, \quad c_2 = 10\left(\frac{1}{25}\right) = \frac{2}{5}, \quad c_3 = 10\left(-\frac{1}{125}\right) = -\frac{2}{25}, \quad c_4 = 10\left(\frac{1}{625}\right) = \frac{2}{125}$

   First four terms: $-2,\ \frac{2}{5},\ -\frac{2}{25},\ \frac{2}{125}$

2. The n^{th} term of a sequence is given. Find the indicated term. Express the term as a fraction in lowest terms. (4)

   $a_n = 6n - 6$; find a_{167}

   Solution: $a_{167} = 6(167) - 6 = 1002 - 6 = 996$

3. Write the first five terms of the sequence defined recursively. Express the terms as simplified fractions when applicable. (4)

   $d_1 = 10,\ d_n = -\dfrac{1}{d_{n-1}}$

   Solution: $d_1 = 10, \quad d_2 = -\frac{1}{10}, \quad d_3 = -\dfrac{1}{-1/10} = 10, \quad d_4 = -\frac{1}{10}, \quad d_5 = 10$

   First five terms: $10,\ -\frac{1}{10},\ 10,\ -\frac{1}{10},\ 10$

Points on this page: /15

4. Evaluate the expression. (3)

   $\dfrac{11!}{4! \cdot 7!}$

Solution: $\dfrac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{4! \cdot 7!} = \dfrac{7920}{24} = 330$

5. Evaluate the expression. (3)

$\dfrac{(2n+3)!}{(2n+4)!}$

Solution: $\dfrac{(2n+3)!}{(2n+4)(2n+3)!} = \dfrac{1}{2n+4}$

6. The n^{th} term of a sequence is given. Find the indicated term. Express the term as a simplified fraction. (3)

$d_n = \dfrac{(n+4)!}{4^n}$; find d_3

Solution: $d_3 = \dfrac{7!}{4^3} = \dfrac{5040}{64} = \dfrac{315}{4}$

7. Find the sum. Express the sum as a fraction in lowest terms. (4)

$\sum_{n=2}^{6} \left(\dfrac{1}{3}\right)^n$

Solution: $\dfrac{1}{9} + \dfrac{1}{27} + \dfrac{1}{81} + \dfrac{1}{243} + \dfrac{1}{729} = \dfrac{81+27+9+3+1}{729} = \dfrac{121}{729}$

Points on this page: /10

8. Write the sum using summation notation. There may be multiple representations. Use i as the index of summation. (4)

$-\frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \frac{1}{256}$

Solution: The terms are $\left(-\frac{1}{4}\right)^1, \left(-\frac{1}{4}\right)^2, \left(-\frac{1}{4}\right)^3, \left(-\frac{1}{4}\right)^4$, so

$$\sum_{i=1}^{4}\left(-\frac{1}{4}\right)^i \qquad \left(\text{equivalently } \sum_{i=1}^{4}\frac{(-1)^i}{4^i}\right)$$

9. Determine whether the sequence is arithmetic. If so, find the common difference. (3)

2, 4, 8, 16, ...

Solution: Differences: $4-2=2$, $8-4=4$, $16-8=8$. The differences are not constant, so the sequence is **not arithmetic**. (It is geometric with $r=2$.)

10. Perform the following:

(a) Write a nonrecursive formula for the n^{th} term of the arithmetic sequence based on the given information. Write numbers as integers or simplified fractions. $a_1 = \frac{1}{4}$, $d = \frac{1}{5}$ (3)

Solution: $a_n = a_1 + (n-1)d = \frac{1}{4} + (n-1)\frac{1}{5} = \frac{1}{5}n - \frac{1}{5} + \frac{1}{4}$

$a_n = \frac{1}{5}n + \frac{1}{20}$

(b) Find a_{10}. Write numbers as integers or simplified fractions. (2)

Solution: $a_{10} = \frac{1}{5}(10) + \frac{1}{20} = 2 + \frac{1}{20} = \frac{41}{20}$

Points on this page: /12

11. Find the sum. (5)

$3 + 8 + 13 + ... + 58$

Solution: Arithmetic with $a_1 = 3$, $d = 5$. Number of terms: $58 = 3 + (n-1)5 \Rightarrow 55 = 5(n-1) \Rightarrow n = 12$.

$S_{12} = \frac{n(a_1 + a_n)}{2} = \frac{12(3+58)}{2} = 6(61) = 366$

12. Write the first five terms of a geometric sequence based on the given information about the sequence. Express the terms as integers or simplified fractions. (5)

$a_1 = 59$ and $a_n = \frac{1}{2}a_{n-1}$ for $n \geq 2$

Solution: $r = \frac{1}{2}$: $a_1 = 59$, $a_2 = \frac{59}{2}$, $a_3 = \frac{59}{4}$, $a_4 = \frac{59}{8}$, $a_5 = \frac{59}{16}$

13. Find the sixth term of the geometric sequence from the given information. Express the term as an integer or simplified fraction. (4)

$a_1 = 16$ and $a_2 = -8$

Solution: $r = \frac{a_2}{a_1} = \frac{-8}{16} = -\frac{1}{2}$

$a_6 = a_1 r^5 = 16\left(-\frac{1}{2}\right)^5 = 16\left(-\frac{1}{32}\right) = -\frac{1}{2}$

Points on this page: /14

14. Find a_1 and r for a geometric sequence from the given information. (5)

$a_2 = 27$ and $a_5 = 729$

Solution: $\frac{a_5}{a_2} = \frac{a_1 r^4}{a_1 r} = r^3 = \frac{729}{27} = 27 \Rightarrow r = 3$
$a_1 = \frac{a_2}{r} = \frac{27}{3} = 9$
$a_1 = 9, \quad r = 3$

15. Find the sum of the geometric series, if possible. (4)

$\sum_{n=1}^{6} 3(3)^{n-1}$

Solution: $a_1 = 3,\ r = 3,\ n = 6$: $\quad S_6 = \frac{a_1(1-r^6)}{1-r} = \frac{3(1-729)}{1-3} = \frac{3(-728)}{-2} = 1092$

16. For the points $(-1, 4)$ and $(4, 9)$,

   (a) Find the exact distance between the points. (3)

   Solution: $d = \sqrt{(4-(-1))^2 + (9-4)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$

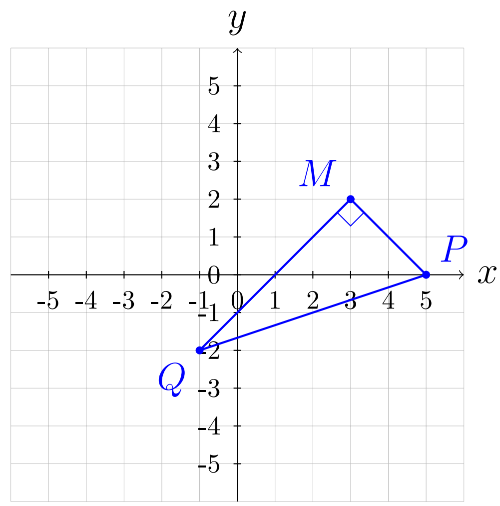
   (b) Find the midpoint of the line segment whose endpoints are the given points. (2)

   Solution: $M = \left(\frac{-1+4}{2}, \frac{4+9}{2}\right) = \left(\frac{3}{2}, \frac{13}{2}\right)$

Points on this page: /14

17. Determine if the given points form the vertices of a right triangle. (5)

$M(3,2)$, $P(5,0)$, and $Q(-1,-2)$



Solution: $MP^2 = (5-3)^2 + (0-2)^2 = 4+4 = 8$ $MQ^2 = (-1-3)^2 + (-2-2)^2 = 16+16 = 32$

$PQ^2 = (-1-5)^2 + (-2-0)^2 = 36+4 = 40$

Since $MP^2 + MQ^2 = 8 + 32 = 40 = PQ^2$, the Pythagorean theorem holds. **Yes**, the points form a right triangle, with the right angle at M.

18. Determine the center and radius of the circle. (3)

$(x-2)^2+(y-7)^2=9$

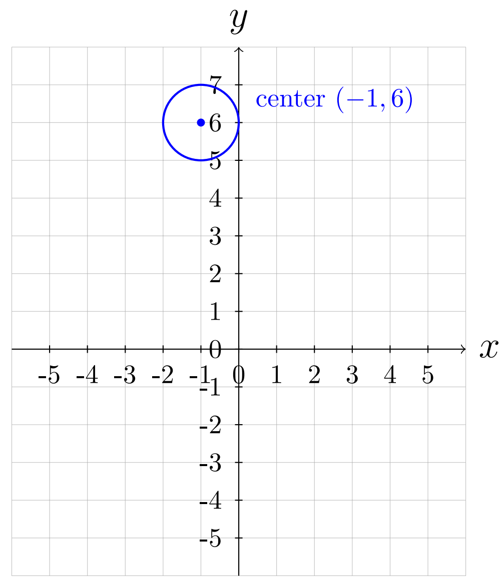
Solution: Center: $(2,7)$ Radius: $r=\sqrt{9}=3$

19. Given a circle with center $(-1,6)$ and radius 1,

(a) Write an equation of the circle in standard form. (3)

Solution: $(x-(-1))^2+(y-6)^2=1^2 \Rightarrow (x+1)^2+(y-6)^2=1$

(b) Graph the circle. (2)



20. Write the equation in the form $(x-h)^2+(y-k)^2=r^2$. Then, if the equation represents a circle, identify the center and radius. (5)

$x^2+y^2+14x-2y+49=0$

Solution: $(x^2+14x+49)+(y^2-2y+1)=-49+49+1$

$(x+7)^2+(y-1)^2=1$

This is a circle with center $(-7,1)$ and radius $r=1$.

Points on this page: /13

—————- Q21 —————- (a)

Solution: Divide both sides by -100:

$$\frac{-100x^2}{-100} - \frac{4y^2}{-100} = \frac{-100}{-100} \quad \Rightarrow \quad \frac{x^2}{1} + \frac{y^2}{25} = 1$$

Center: $(0, 0)$

(b)

Solution: $25 > 1$ and 25 is under y^2, so the major axis is vertical.
$a^2 = 25 \Rightarrow a = 5, \qquad b^2 = 1 \Rightarrow b = 1$

(c)

Solution: Vertices: $(0, \pm a) = (0, 5)$ and $(0, -5)$

(d)

Solution: Endpoints of the minor axis: $(\pm b, 0) = (1, 0)$ and $(-1, 0)$

(e)

Solution: $c^2 = a^2 - b^2 = 25 - 1 = 24 \Rightarrow c = \sqrt{24} = 2\sqrt{6}$
Foci: $(0, 2\sqrt{6})$ and $(0, -2\sqrt{6})$

—————- Q22 —————-

Solution: $\dfrac{(x-(-1))^2}{36} + \dfrac{(y-3)^2}{49} = 1$, so $h = -1$, $k = 3$.
(b) $(-1, 3)$

—————- Q23 —————-

Solution: Group terms and move the constant:

$$(9x^2 - 36x) + 25y^2 = -35$$

Factor out 9 and complete the square $\left(\left(\frac{1}{2}\cdot(-4)\right)^2 = 4\right)$:

$$9(x^2 - 4x + 4) + 25y^2 = -35 + 9(4) \quad \Rightarrow \quad 9(x-2)^2 + 25y^2 = 1$$

Write the coefficients as denominators:

$$\frac{(x-2)^2}{\frac{1}{9}} + \frac{y^2}{\frac{1}{25}} = 1$$

—————- Q24 —————-

Solution: The vertices have the same y-coordinate, so the transverse axis is horizontal.

Center (midpoint of vertices): $\left(\dfrac{4+(-6)}{2}, \dfrac{2+2}{2}\right) = (-1, 2)$

a = distance from center to a vertex $= |4-(-1)| = 5$

Slope of asymptotes: $\pm\dfrac{b}{a} = \pm\dfrac{7}{5} \Rightarrow \dfrac{b}{5} = \dfrac{7}{5} \Rightarrow b = 7$

$$\frac{(x+1)^2}{25} - \frac{(y-2)^2}{49} = 1$$

—————- Q25 —————-

Solution: Rewrite: $\dfrac{(y+3)^2}{36} - \dfrac{(x+2)^2}{49} = 1$

(a) The terms have opposite signs, so **the equation is a hyperbola**.

(b) The positive term contains y, so the transverse axis is vertical. Center $(-2,-3)$, $a = 6$, $b = 7$. Vertices: $(-2, -3 \pm 6) = (-2, 3)$ and $(-2, -9)$. The branches open up and down, so the correct graph is **A**.

—————- Q26 —————-

Solution: From the graph, the branches open left and right, so the transverse axis is horizontal.

Center: $(0,2)$ Vertices: $(-2,2)$ and $(2,2) \Rightarrow a = 2$

The reference rectangle extends from $y = 1$ to $y = 3 \Rightarrow b = 1$

(Check: asymptote slopes $\pm\frac{b}{a} = \pm\frac{1}{2}$ match the dashed lines.)

$$\frac{x^2}{4} - \frac{(y-2)^2}{1} = 1$$

—————- Q27 —————- (a)

Solution: $y^2 = -11x$, so $4p = -11 \Rightarrow p = -\frac{11}{4}$

(b)

Solution: Focus: $(p,0) = \left(-\frac{11}{4}, 0\right)$

(c)

Solution: Directrix: $x = -p \Rightarrow x = \frac{11}{4}$