Math 140: Tuesday, Nov 12th, Day 20 Notes

Agenda for Tuesday, Nov 12th, 2024	
Start Logic Chapter 5	**1**

Coming up

- ! Online HW 12 due today
- ! Exam 3 next week on Thursday (material: starting at Day 15 - counting methods and including everything up until the exam)
- Deadline to withdraw from class is today

Statements

A statement is a declarative sentence that is either true or false but not both true and false at the same time. Each statement has a $\underbrace{\text{truth value}}_{T,\ F}$. It is either True or False.

Examples of statements:

1. Barack Obama was the president of the USA. T
2. The number 2 is even and less than 10. T
3. Jimmy Fallon was responsible for the development of graph theory. F

Examples that are not statements:

Liars paradox

1. What is your name? it's a question.
2. Pick that up! a command
3. This statement is false. This is an example of a paradox.

NOT

Compound Statements

A compound statement is a statement that combines statements using logical connectors such as: not, and, or,

if ... then ... .
if and only if.

Examples of compound statements:

1. If you eat your vegetables, then you get a cookie.
2. Tom is taking MTH 140 or Tom is taking MTH 141.
3. I biked to work and I carried my lunch.

Not a compound statement: Phineas and Ferb TV show was funny.

Symbolic Logic has the goal to translate statements into symbols, analyze or simplify them, and then translate them back into words.

We use lower case letters to denote simple statements.

Example:

p: $1+1=2$ T.

=

r: $6<5$ false

q: It's raining outside. F

Truth Tables

A truth table is a table that shows all possibilities for the truth values of a statement.

simple statement

p	r	
T	T	
F	F	

Negation we denote the negation of a statement p with ~p.

The negation of a true statement is <u>false</u> and the negation of a false statement is <u>true</u>. (The statement and its negation make up all the possibilities).

Examples – Write the negation of each statement:

- p: My car is a Ford.

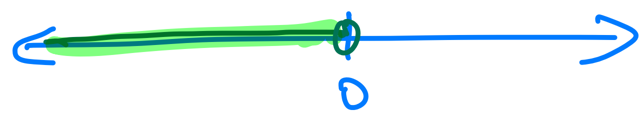
~p: My car is not a Ford.

- q: Today is not Thursday. T

~q: Today is Thursday. F

- r: $6x < 0$

~r : $6x \geq 0$



- s: $y \geq 19$

~s: $y < 19$

$>$ negate $\leq$

$\leq$ negate $>$

Quantifiers

Words such as <u>all</u> (universal quantifier), <u>some</u>, <u>none</u>, ~~___~~ are called quantifiers. (existential quantifier)

Note: The word "some" in this context means <u>at</u> <u>least</u> <u>one</u>.

Examples of statements with quantifiers - Write the negation of each statement:

All politicians lie. T

~(All politicians lie): F

p: 1. All cars have airbags.

~p: Some cars don't have airbags

p: 2. Some cars have antilock brakes.

~p: All cars don't have antilock brakes.

or

No cars have antilock brakes.

p: 3. No cars have window stickers. p: All cars don't have window stickers

~p: Some cars have window stickers.

p: 4. Some cars do not have air conditioning. T

~p: All cars have A.C. F

p: 5. At least one car has a bumper sticker. T

~p: All cars don't have bumper sticker. F

or No cars have bumper stickers.

Summary

p

all are (has)

some are (have)

$\sim p$

Some are not (don't have)

all are not (don't have)

$\mathbb{Q} \subseteq \mathbb{N}$? F

$\boxed{\mathbb{N} \subseteq \mathbb{Q} \quad T}$

$\frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}$

Irrational numbers $= \mathbb{R} \setminus \mathbb{Q}$

Sets of numbers:

$\mathbb{N}$ = set of natural numbers $= \{1, 2, 3, \ldots\}$

W = set of whole numbers $= \{0, 1, 2, 3, \ldots\} = \mathbb{N} \cup \{0\}$

$\mathbb{Z}$ = set of integers $= \{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$

$\mathbb{Q}$ = set of rational numbers $= \{\frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0\}$

$\in$ is a member of

ex: $\frac{1}{2} \in \mathbb{Q}$ T $\quad \frac{-2}{3} \in \mathbb{Q}$ T

$\pi \notin \mathbb{Q}$

$\mathbb{R}$ = real numbers $= \{x \mid x$ can be written as a decimal$\}$

$\pi \in \mathbb{R}$

True or False.

1. Every natural number is an integer. T
2. There exists an integer that is not a natural number. T
3. All irrational numbers are real numbers. T
4. Some whole numbers are not rational numbers. F
5. Each rational number is a positive number. F

1) Some natural numbers are not integers. F
2) All integer are natural numbers. F
3) Some irrational numbers are not real numbers F.
4) All whole numbers are rational numbers. . T
5) Some rational numbers are not positive numbers . T

Conjunction and Disjunction

next time

We can combine statements using "and" and "or".
Conjunction: "and"

Symbol:

Disjunction: (inclusive "or" meaning A or B or both)

Symbol:

Example: Let p: The dog is large. q: The dog is fluffy.

Translate from symbols to words:

1. $\neg q$

2. $\neg(q)$

3. $p \vee q$

4. $p \wedge q$

5. $\neg(p \vee q)$

6. $\neg(p \wedge q)$

Translate from words to symbols:

1. The dog is large or the dog is not fluffy.
2. The dog is neither large nor fluffy.