

Section 4.4 Laws/Properties of Logs

Properties of Logarithms with base b , where $b, x, y > 0, b \neq 1$, and p is a real number

- | | | |
|-----------------------|--|--------------------------|
| 1. $\log_b 1 = 0$ | 5) $\log_b(x \cdot y) = \log_b x + \log_b y$ | <u>product</u> property |
| 2. $\log_b b = 1$ | 6) $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$ | <u>quotient</u> property |
| 3. $\log_b b^x = x$ | 7) $\log_b(x^p) = p \cdot \log_b x$ | <u>power</u> property |
| 4. $b^{\log_b x} = x$ | | |

Condensing Logarithmic Expressions

Write each expression as a single logarithm.

- $\log_3(4) + \log_3(5)$
- $\log_2(9) - \log_2(3)$
- $2 \log(x) - \log(y) + \log(z)$
- $\log_5(3) + \log_5(x^2) - \log_5(7)$
- $\frac{1}{2} \log_6(x) + 3 \log_6(y) - \log_6(z^2)$
- $\log_b(x^m y^n) - \log_b(z^p) + \frac{1}{q} \log_b(w^r)$

Expanding Logarithmic Expressions

Write each expression as a sum or difference of logarithms.

1. $\log_4(12)$

4. $\log_5 \left(\frac{2^3 x^4}{y^2 z} \right)$

2. $\log_7 \left(\frac{a}{b} \right)$

5. $\log \left(\frac{(ab)^3 \sqrt[3]{c}}{\sqrt{d^5}} \right)$

3. $\log_3(5x^2y)$

6. $\log_b \left(\sqrt[n]{\frac{x^m y^p}{z^q w^r}} \right)$

Section 3.4 Solving Exponential and Logarithmic Equations

1. $5^{2x} = 3$

4. $e^{x^2+x} = e^{3x+15}$

2. $160e^{-0.03t} = 80$

5. $(1 + 0.05/4)^{4t} = 3$

3. $96 - 10^x = 12$

Solving Exponential Equations

- isolate the exponential expression
- take the log of both sides
- simplify using log properties
- solve the equation by isolating the variable

EX Solve the logarithmic equation algebraically.

1. $\log(2x + 4) = \log(3 - x)$

2. $\ln(x) + \ln(x - 3) = \ln(5x - 7)$

3. $2\log_5 x = \log_5(7) + \log_5(3x)$

Solving log equations (all terms have a log with the same base)

1. condense logs into a single log (first use prop 7), if multiple logs on one side
2. solve the equation using the arguments
3. check and discard extraneous solutions

4. $-3 + 4 \ln(7x) = 2$

5. $\ln(\sqrt{x-8}) = 5$

6. $3 \log_2(x+4) = 7$

7. $\log_4(x+2) = 1 - \log_4(x-1)$

8. $\log x = 2 + \log(x-3)$

Solving log equations (has a term without a log)

1. isolate terms with a log of same base to one side
2. condense logs into a single log and isolate
3. write as an exponential equation
4. solve the equation
5. check and discard extraneous solutions