Setion 5.1: Using Fundamental Trigonometric Identities

1 Recall: Fundamental Trigonometric Identities

Type	**Identities**
Reciprocal	$\sin\theta = \frac{1}{\csc\theta}$
	$\cos\theta = \frac{1}{\sec\theta}$
	$\tan\theta = \frac{1}{\cot\theta}$
	$\csc\theta = \frac{1}{\sin\theta}$
	$\sec\theta = \frac{1}{\cos\theta}$
	$\cot\theta = \frac{1}{\tan\theta}$
Quotient	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
	$\cot\theta = \frac{\cos\theta}{\sin\theta}$
Pythagorean	$\sin^2\theta + \cos^2\theta = 1$
	$\boxed{1 + \tan^2\theta = \sec^2\theta}$
	$1 + \cot^2\theta = \csc^2\theta$
Cofunction	$\sin(\frac{\pi}{2} - \theta) = \cos\theta$
	$\cos(\frac{\pi}{2} - \theta) = \sin\theta$
	$\tan(\frac{\pi}{2} - \theta) = \cot\theta$
	$\cot(\frac{\pi}{2} - \theta) = \tan\theta$
	$\sec(\frac{\pi}{2} - \theta) = \csc\theta$
	$\csc(\frac{\pi}{2} - \theta) = \sec\theta$
Even/Odd	$\sin(-\theta) = -\sin\theta$ (odd)
	$\cos(-\theta) = \cos\theta$ (even)
	$\tan(-\theta) = -\tan\theta$ (odd)
	$\cot(-\theta) = -\cot\theta$ (odd)
	$\sec(-\theta) = \sec\theta$ (even)
	$\csc(-\theta) = -\csc\theta$ (odd)

Table 1: Fundamental Trigonometric Identities

$\sin^2\theta = \cancel{1} - \cos^2\theta$

$\sin^2\theta - 1 = -\cos^2\theta$

$\sec^2\theta - 1 = \tan^2\theta$

$1 = \csc^2\theta - \cot^2\theta$

$1 - \csc^2\theta = -\cot^2\theta$

2 Example 1: Simplifying Trigonometric Expressions

2.1 Problem 1.1: Simplify $\sin x \cos^2 x - \sin x$

Solution:

$$\sin x\,(\underbrace{\cos^2 x - 1}_{\text{used}}) = \sin x\,(-\sin^2 x)$$
$$= \boxed{-\sin^3 x}$$

$$\sin^2 x + \cos^2 x = 1$$
$$\sin^2 x = 1 - \cos^2 x$$
$$-\sin^2 x = -(1-\cos^2 x) = \cos^2 x - 1$$

2.2 Problem 1.2: Simplify $\boxed{\sin t + \cot t \cos t}$

Solution: $\sin t + \frac{\cos t}{\sin t} \cos t =$

$$= \frac{\sin t}{1} + \frac{\cos^2 t}{\sin t} = \sin t\,\frac{\sin t}{\sin t} + \frac{\cos^2 t}{\sin t}$$

$$= \frac{\sin^2 t + \cos^2 t}{\sin t} = \frac{1}{\sin t} = \boxed{\csc t}$$

3 Example 2: Factoring Trigonometric Expressions

3.1 Problem 2.1: Factor $\boxed{\sec^2\theta - 1}$ $= \sec^2\theta - 1^2$ (difference of squares)

Solution:

$$a^2 - b^2 = (a-b)(a+b)$$

$$a = \sec\theta \ \&\ b = 1$$

$$\sec^2\theta - 1^2 = \boxed{(\sec\theta - 1)(\sec\theta + 1)}$$

quadratic form

3.2 Problem 2.2: Factor $4\tan^2\theta + \tan\theta - 3$

Review AC method for factoring quadratics!

Solution: Let $\tan\theta = x$.

$$4x^2 + x - 3 = (4x-3)(x+1)$$

$$4x^2 + x - 3 = (4x-3)(x+1)$$

$$\boxed{4\tan^2\theta + \tan\theta - 3 = (4\tan\theta - 3)\cdot(\tan\theta + 1)}$$

3.3 Problem 2.3: Factor $\csc^2 x - \cot x - 3$

Solution:

use $\boxed{\csc^2 x = 1 + \cot^2 x}$

$$\csc^2 x - \cot x - 3 = 1 + \cot^2 x - \cot x - 3$$

$$= \boxed{\cot^2 x - \cot x - 2}$$

Let $\cot x = y$.

$$y^2 - y - 2 = (y-2)(y+1)$$

$$= (\cot x - 2)(\cot x + 1)$$

$$\boxed{\csc^2 x - \cot x - 3 = (\cot x - 2)(\cot x + 1)}$$

multiply by conjugate

4 Example 3: Rewriting Trigonometric Expressions

4.1 Problem: Rewrite $\frac{1}{1+\sin x}$

The conjugate of $1+\sin x$ is $1-\sin x$.

Solution:

$$\frac{1}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x} = \frac{1-\sin x}{(1+\sin x)(1-\sin x)} =$$

$$= \frac{1-\sin x}{1 \cancel{-\sin x} \cancel{+\sin x} - \sin^2 x} = \frac{1-\sin x}{1-\sin^2 x}$$

$$= \frac{1-\sin x}{\cos^2 x} = \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} = \sec^2 x - \frac{\sin x}{\cos x \cos x}$$

$$= \sec^2 x - \tan x \sec x$$

5 Example 4: Trigonometric Substitution

$\sqrt{x^2} = x$

5.1 Problem: Use the substitution $x = 2\tan\theta$ (with θ in quadrant I) to rewrite $\sqrt{4+x^2}$

Solution:

$$\sqrt{4+x^2} = \sqrt{4+(2\tan\theta)^2} = \sqrt{4+4\tan^2\theta}$$

$$= \sqrt{4(1+\tan^2\theta)} = \sqrt{4}\sqrt{1+\tan^2\theta}$$

$$= 2\sqrt{\sec^2\theta}$$

$$= 2\sec\theta$$

$$* \boxed{\int \sqrt{4+x^2}\,dx = \int 2\sec\theta\, d\theta}$$

$$\sqrt{\sec^2\theta} = |\sec\theta|$$

$$= \pm\sec\theta$$

since θ is in QI, $\sec\theta = \frac{1}{\cos\theta}$ is positive.