

MAC 1140 : Precalc
Algebra
12.1 Sequences and Summation
Notation

1. Sequences

$$\text{ex: } \{-1, 1, -1, 1, -1, 1, \dots\}$$

In this section we are going to introduce Sequences and Summation Notation. Roughly speaking, a sequence is an infinite list of numbers. The numbers in the sequence are often written as $a_1, a_2, a_3, \dots$. The dots mean that the list continues forever.

A simple example is the sequence:

$$\{5, 10, 15, 20, 25, \dots\} \quad a_n = 5 \cdot n \quad a_{2020} = 5(2020)$$

$n=1 \quad a_1=5 \quad n=2 \quad a_2=10 \quad n=3 \quad a_3=15 \quad 5 \cdot 4 \quad 5 \cdot 5$

We can describe the pattern of the sequence displayed above by the following formula:

$$a_n = 5n$$

general term

Example 1: Find the first four terms and the 100th term of the sequence defined by each formula.

① $a_n = 2n - 1$

2. $b_n = n^2 - 1$

3. $c_n = \frac{n}{n+1}$

4. $d_n = \frac{(-1)^{n+1}}{n}$

① $a_n = 2n - 1$

$a_1 = 2 \cdot 1 - 1 = 1$

$a_2 = 2 \cdot 2 - 1 = 3$

$a_3 = 2 \cdot 3 - 1 = 5$

$a_4 = 2 \cdot 4 - 1 = 7$

$\vdots$
 $a_{100} = 2 \cdot 100 - 1 = 199$

$$\{1, 3, 5, 7, \dots\}$$

$$\sum_{n=1}^{\infty} (2n-1) = a_1 + a_2 + a_3 + \dots = 1 + 3 + 5 + 7 + \dots$$

② $b_n = n^2 - 1$

$b_1 = 1^2 - 1 = 0$

$b_2 = 2^2 - 1 = 3$

$b_3 = 3^2 - 1 = 8$

$b_4 = 4^2 - 1 = 15$

$b_{100} = 100^2 - 1$

$\{0, 3, 8, 15, \dots\}$

③ $c_n = \frac{n}{n+1}$

$c_1 = \frac{1}{1+1} = \frac{1}{2}$

$c_2 = \frac{2}{2+1} = \frac{2}{3}$

$c_3 = \frac{3}{3+1} = \frac{3}{4}$

$c_4 = \frac{4}{4+1} = \frac{4}{5}$

$c_{100} = \frac{100}{101}$

④ $d_n = \frac{(-1)^{n+1}}{n}$ $\{1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \dots\}$

$d_1 = \frac{(-1)^{1+1}}{1} = +1$

$d_2 = \frac{(-1)^{2+1}}{2} = -1/2$

$d_3 = \frac{(-1)^{3+1}}{3} = +1/3$

$d_4 = \frac{(-1)^{4+1}}{4} = -1/4$

$d_{100} = \frac{(-1)^{100+1}}{100} = -\frac{1}{100}$

Example 2: Find the n th term of a sequence whose first several terms are given.

(a) $\left\{ \frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \dots \right\}$

(b) $-2, 4, -8, 16, -32, \dots$

$$a_1 = \frac{1}{2} \quad [n=1] = \frac{2 \cdot 1 - 1}{2 \cdot 1}$$

$$a_2 = \frac{3}{4} \quad [n=2] = \frac{2 \cdot 2 - 1}{2 \cdot 2}$$

$$a_3 = \frac{5}{6} \quad [n=3] = \frac{2 \cdot 3 - 1}{2 \cdot 3}$$

$$a_n = \frac{2n-1}{2n}$$

$$(b) a_1 = -2 = (-1)^1 \cdot 2 = (-1)^1 2^1$$

$$a_2 = 4 = (-1)^2 \cdot 4 = (-1)^2 2^2$$

$$a_3 = -8 = (-1)^3 \cdot 8 = (-1)^3 2^3$$

$$a_4 = 16 = (-1)^4 \cdot 16 = (-1)^4 2^4$$

$$a_n = ? \quad a_n = (-1)^n \cdot 2^n$$

2. Recursively defined sequences

Some sequences do not have simple defining formulas like those of the preceding example. The n th term of a sequence may depend on some or all of the terms preceding it. A sequence defined in this way is called recursive.

Example 3 - The Fibonacci Sequence:

Find the first 7 terms of the the sequence defined recursively by

$$F_1 = 1, F_2 = 1 \text{ and } F_n = F_{n-1} + F_{n-2}.$$

$$F_1 = 1$$

$$F_2 = 1$$

$$F_3 = F_2 + F_1 = 1 + 1 = 2$$

$$F_4 = F_3 + F_2 = 2 + 1 = 3$$

$$F_5 = F_4 + F_3 = 3 + 2 = 5$$

$$F_6 = F_5 + F_4 = 5 + 3 = 8$$

$$F_7 = F_6 + F_5 = 8 + 5 = 13$$

$$[Ex] a_1 = 2$$

$$a_2 = 2 + 3$$

$$a_2 = a_1 + 3$$

$$a_3 = a_2 + 3$$

$\vdots$

$$[a_n = a_{n-1} + 3]$$

$$\{1, 1, 2, 3, 5, 8, 13, 21, \dots\}$$

3. Partial sums and Sigma notation

If we add the terms of a sequence we obtain a series. So a series is an infinite sum of numbers which may or may not add up to an actual value. In calculus 2 you will learn how to find what an infinite series adds up to for certain types of series.

A finite series is a sum of numbers, which can simply be added up to a total. For both infinite series and finite series we use Sigma notation.

$$\sum_{n=1}^{\infty} a_n$$

sequence: $a_1, a_2, a_3, a_4, \dots$

series $= a_1 + a_2 + a_3 + a_4 + \dots = \sum_{n=1}^{\infty} a_n$

$$a_1 + a_2 + a_3 + a_4 + \dots = S$$

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sequence $\{1, -1/2, 1/3, -1/4, \dots\}$

series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots = \ln 2$

A finite series/partial sum

sequence: $a_1, a_2, a_3, a_4, \dots$

$$a_1 + a_2 + a_3 = \text{finite sum} = \text{partial sum} = S_3 = \sum_{n=1}^3 a_n$$

$$a_1 + a_2 + a_3 + a_4 + a_5 = S_5 = \sum_{n=1}^5 a_n$$

I can find the sum of partial sums always.

$$a_n = 3n + 1$$

$$S_3 = a_1 + a_2 + a_3 = (3 \cdot 1 + 1) + (3 \cdot 2 + 1) + (3 \cdot 3 + 1) = 4 + 7 + 10 = 21$$