Math 140: Chapter 3 Probability

Consider a standard deck of cards.

2♡	3♡	4♡	5♡	6♡	7♡	8♡	9♡	10♡	J♡	Q♡	K♡	A♡
2♢	3♢	4♢	5♢	6♢	7♢	8♢	9♢	10♢	J♢	Q♢	K♢	A♢
2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣	Q♣	K♣	A♣
2♠	3♠	4♠	5♠	6♠	7♠	8♠	9♠	10♠	J♠	Q♠	K♠	A♠

clubs

Spades

$P(5\heartsuit) = \frac{1}{52}$; $P(\text{face card}) = \frac{12}{52}$

3.2 Probability involving compound events

Often we deal with events that are described in terms of other events. These are called compound events and they involve the connectors *or*, *and* and *not*.

Probabilities involving NOT

Complement Rule: (not)

$P(A) \rightarrow$ means probability of event A happening

$P(\text{not } A)$ also written as $P(A')$ ($P(A^c)$), $P(\bar{A})$)

mean probability of event A not happenning

$$\boxed{P(\text{not } A) = 1 - P(A)}$$

$P(5\heartsuit) = \frac{1}{52}$

$P(\text{not } 5\heartsuit) = \frac{51}{52}$

$P(\text{not } 5\heartsuit) = 1 - P(5\heartsuit) = 1 - \frac{1}{52} = \frac{51}{52}$

$P(A \text{ or } B) = \frac{9}{12}$ prob of rolling an even number or Tail on coin.

Probabilities involving OR

Example 1. Consider the experiment of rolling a six sided die and flipping a coin. $n(S) = 12$

Sample space: $S = \{1T, 1H, 2T, 2H, 3T, 3H, 4T, 4H, 5T, 5H, 6T, 6H\}$.

Let event A be the event of rolling an even number and event B be the event of getting Tail on the coin.

$A = \{2T, 2H, 4T, 4H, 6T, 6H\}$ $\boxed{n(A) = 6}$

$B = \{1T, 2T, 3T, 4T, 5T, 6T\}$ $\boxed{n(B) = 6}$

The event A OR B includes

$\boxed{A \text{ or } B = \{1T, 2T, 3T, 4T, 5T, 6T, 2H, 4H, 6H\}}$

combine events A and B with no duplicates $\boxed{n(A \text{ or } B) = 9}$

Mutually exclusive events are events that have no outcomes in common (or they can't happen at the same time).

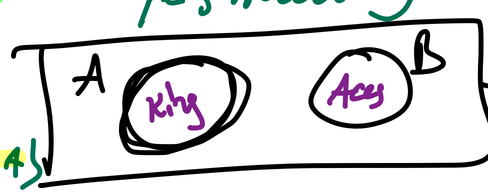
Example 2. Are the following events mutually exclusive?

1. Let A be the event of "drawing a King" out of a deck of cards and B be the event of "drawing an Ace". yes mutually exclusive

$A = \{K♡, K◇, K♤, K♧\}$

$B = \{A♡, A◇, A♤, A♧\}$

$A \text{ or } B = \{K♡, K◇, K♤, K♧, A♡, A◇, A♤, A♧\}$



sample space

2. Let A be the event of "drawing a King" out of a deck of cards and B be the event of "drawing a Heart".

No

$P(K \text{ or } ♡)$
$= P(K) + P(♡) - P(K♡)$
$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$
$= \frac{4+13-1}{52} = \frac{16}{52} = \frac{4}{13}$

3. Let A be the event of "getting an A" and B the event of "getting a B" in this class.

yes !

Addition Rule for mutually exclusive events:

For two mutually exclusive events A and B, the probability that one **or** the other occurs is the sum of the probabilities of the two events.

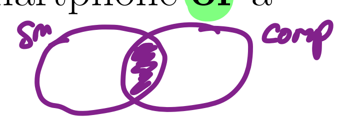
$$\boxed{P(A \text{ or } B) = P(A) + P(B)}$$

$$P(\text{King or Ace}) = P(\text{King}) + P(\text{Ace}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

The general additional rule

The Addition Rule doesn't work for events that aren't mutually exclusive. If the probability of owning a smartphone is 0.50 and the probability of owning a computer is 0.90, the probability of owning a smartphone **or** a computer may be pretty high, but it is not 1.40!

$$P(A \text{ or } B) = P(A) + P(B) = 0.5 + 0.9 = 1.4 \quad ✗$$



We'd like to generalize the Addition Rule to include events that aren't mutually exclusive.

General Addition Rule: A and B can be mutually exclusive or not.

For any events A and B, the probability that A **or** B occurs is the sum of the probabilities of the two events minus the probability that both A and B occur.

$$\boxed{P(A \text{ or } B) = P(A) + P(B) - \underbrace{P(A \text{ and } B)}_{\text{overlap}}}$$

revisit ex 2: A = event of King
B = event of A

$= 0$

P(R) = prob of being relationg = .33 , P(S) = prob of sports = .25

Example 3. A survey of college students asked the questions: "Are you currently in a relationship?" and "Are you involved sports?" The survey found that 33% were currently in a relationship, and 25% were involved in sports. Eleven percent responded "yes" to both.

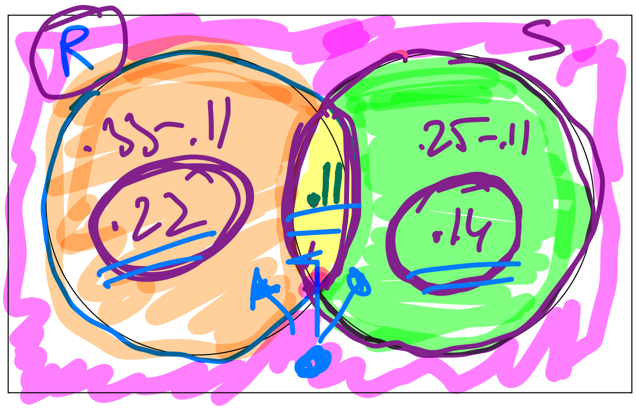
$P(R \text{ and } S) = .11$

1. We can use a Venn Diagram to answer the following questions.

$P(R) = 0.33$

$\boxed{P(S) = 0.25}$

$P(R \text{ and } S) = 0.11$



What is the probability that a randomly selected student:

(a) is in a relationship **or** is involved in sports?

(b) is not in a relationship and is not involved in sports? $1 - .47 = .53$

a) sol 1: $P(R \text{ or } S) = P(R) + P(S) - P(R \text{ and } S) = .33 + .25 - .11 = .47$

sol 2: $.22 + .11 + .14 = .47$

(c) is in a relationship but not involved in sports?

.22

(d) either is in a relationship or is involved in sports, but not both?

$.22 + .14 = .36$

only rel — only sports