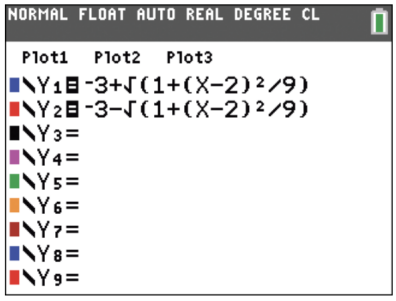
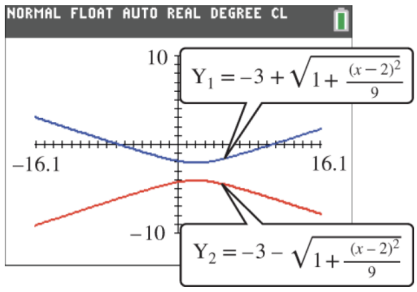
Calculator Instructions:





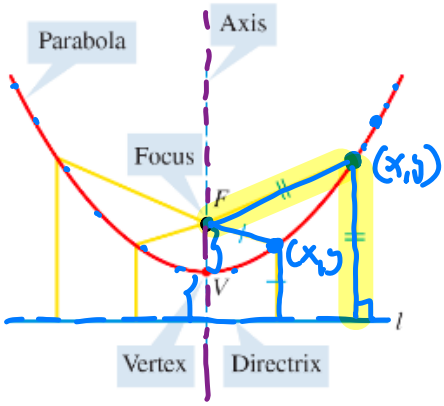
$y = ax^2 + bx + c$

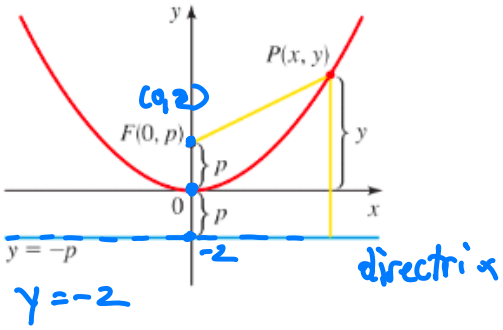
5. 9.1 Continued: Parabolas

In this section we study parabolas from a geometric rather than an algebraic point of view. We will start with parabolas that are situated with the vertex at the origin and that have a vertical or horizontal axis of symmetry.

GEOMETRIC DEFINITION OF A PARABOLA

A **parabola** is the set of all points in the plane that are equidistant from a fixed point F (called the **focus**) and a fixed line l (called the **directrix**).





PARABOLA WITH VERTICAL AXIS

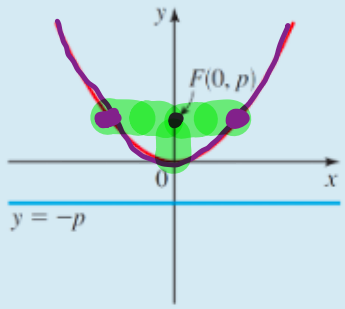
The graph of the equation

$$x^2 = 4py$$

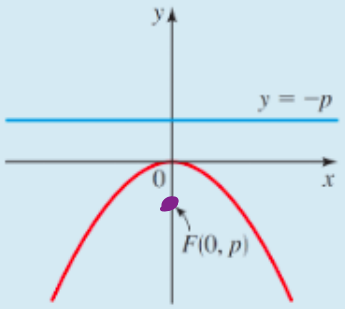
is a parabola with the following properties.

VERTEX	$V(0, 0)$
FOCUS	$F(0, p)$
DIRECTRIX	$y = -p$

The parabola opens upward if $p > 0$ or downward if $p < 0$.



$x^2 = 4py$ with $p > 0$



$x^2 = 4py$ with $p < 0$

PARABOLA WITH HORIZONTAL AXIS

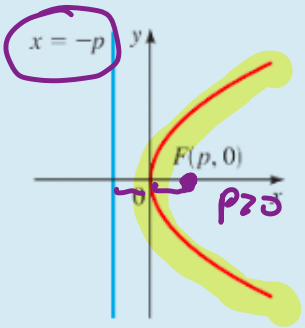
The graph of the equation

$$y^2 = 4px$$

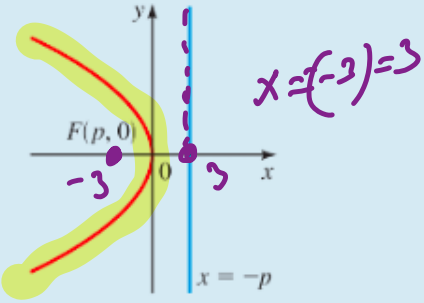
is a parabola with the following properties.

VERTEX	$V(0, 0)$
FOCUS	$F(p, 0)$
DIRECTRIX	$x = -p$

The parabola opens to the right if $p > 0$ or to the left if $p < 0$.



$y^2 = 4px$ with $p > 0$

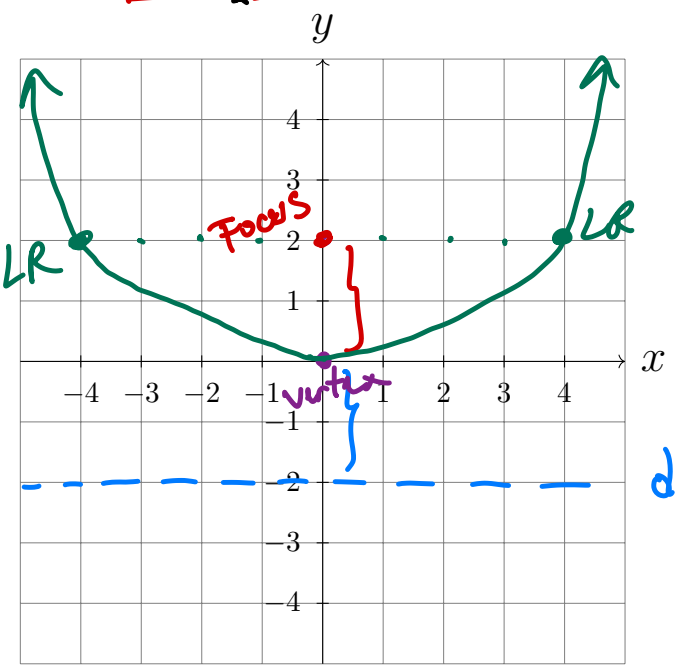


$y^2 = 4px$ with $p < 0$

Lactus rectum endpoints: are the points on the parabola that $2|p|$ units from the focus.

Note: The parabola always opens towards the focus and away from the directrix!!!

Example 1: Find an equation for the parabola with vertex $V(0,0)$ and focus $F(0,2)$, and sketch its graph.



equation:

$x^2 = 4py$

$p = 2 > 0$ opens up

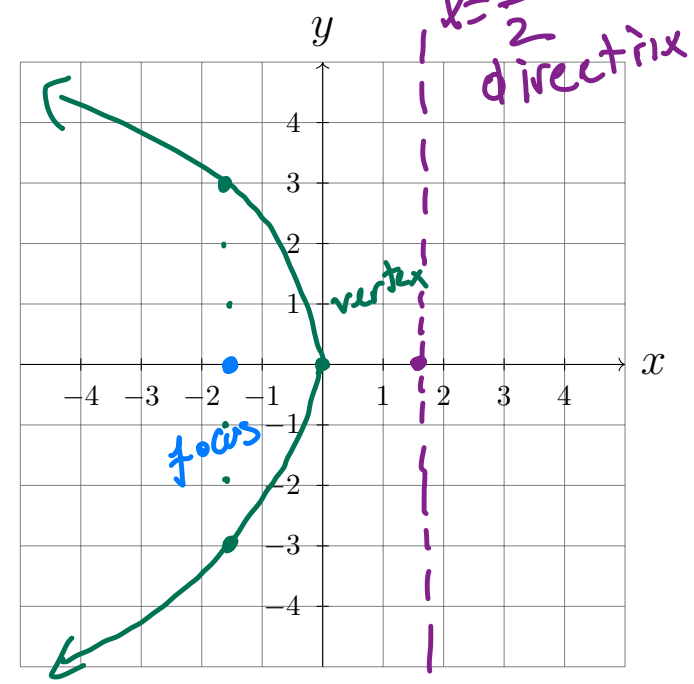
Loctus rectum:

$2|p| = 2(2) = 4$ units from the focus.

Ans: $x^2 = 4 \cdot 2y \Rightarrow x^2 = 8y$

$y = \frac{1}{8}x^2$

Example 2: A parabola has the equation $6x + y^2 = 0$.
Find the focus, focal diameter and directrix of the parabola and draw the graph.



$6x + y^2 = 0$

$y^2 = -6x$

$y^2 = 4px$

Solve: $4p = -6$

$p = -\frac{6}{4}$ $\Rightarrow p = -\frac{3}{2}$

parabola opens to the left.

need LR $2|p| = 2\left|-\frac{3}{2}\right| =$

$= 2 \cdot \frac{3}{2} = 3$

focal diameter: $4|p| = 4\left(\frac{3}{2}\right) = 6$

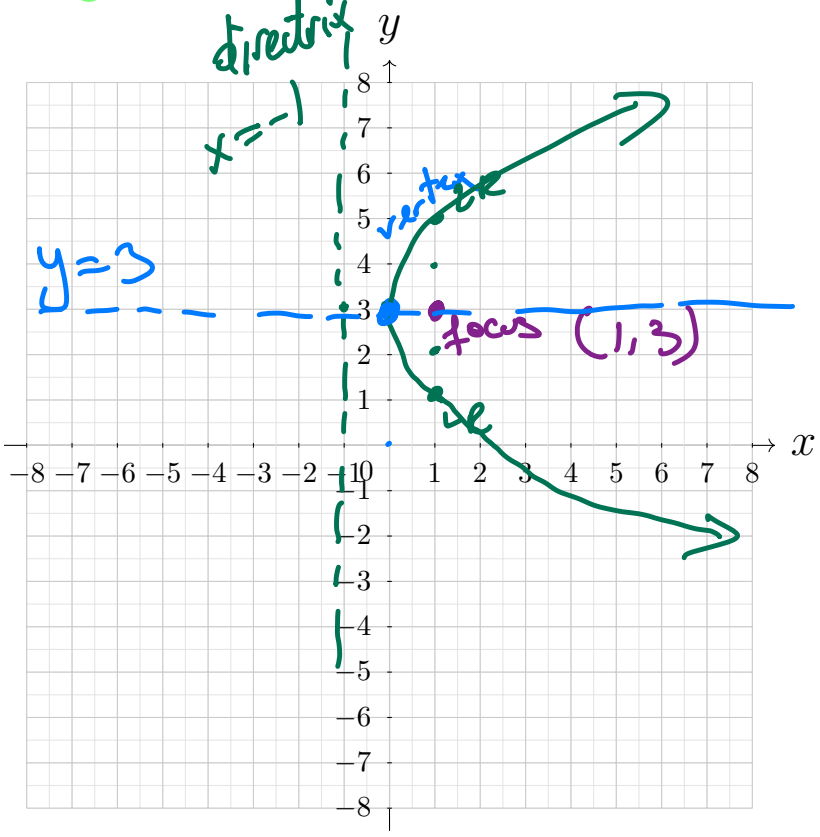
6. Graphing hyperbolas with different center → vertex

The following summarizes the equation and features of hyperbolas centered at (h, k).

$x^2 = 4py$ → (h,k) → $(x-h)^2 = 4p(y-k)$ ×

$y^2 = 4px$ → $(y-k)^2 = 4p(x-h)$ ×

Example 3: Identify the vertex, focus, directrix, axis of symmetry, lactus rectum of the parabola $y^2 - 6y - 4x + 9 = 0$. Draw the parabola.



$y=3$

$y^2-6y-4x+9=0$

$y^2-6y = 4x-9$

$y^2-6y+9 = 4x-9+9$

$(y-3)^2 = 4(x-0)$

$(h,k) = (0,3)$ vertex

$p = 1 > 0$ opens to the right.

LR : $2|p| = 2|1| = 2$