Calculus 2 Prep: Derivative & Integral Review

A quick refresher before we dive into Calc 2

Basic Derivatives

Power & Constant

$$(c)' = 0$$
$$(x^n)' = nx^{n-1}$$

Exponential & Logarithmic

$$(e^x)' = e^x$$
$$(a^x)' = a^x \ln a$$
$$(\ln x)' = \frac{1}{x}$$
$$(\log_a x)' = \frac{1}{x \ln a}$$

Trigonometric

$$(\sin x)' = \cos x$$
$$(\cos x)' = -\sin x$$
$$(\tan x)' = \sec^2 x$$
$$(\cot x)' = -\csc^2 x$$
$$(\sec x)' = \sec x \tan x$$
$$(\csc x)' = -\csc x \cot x$$

Inverse Trigonometric

$$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$$
$$(\cos^{-1} x)' = \frac{-1}{\sqrt{1-x^2}}$$
$$(\tan^{-1} x)' = \frac{1}{1+x^2}$$

Differentiation Rules

Constant multiple: $[cf(x)]' = cf'(x)$

Sum/Difference: $(f \pm g)' = f' \pm g'$

Product rule: $(fg)' = f'g + fg'$

Quotient rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

Chain rule: $[f(g(x))]' = f'(g(x))\, g'(x)$

Basic Integrals

Power & Constant

$$\int k\,dx = kx + C$$
$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$$
$$\int \frac{1}{x}\,dx = \ln|x| + C$$

Exponential

$$\int e^x\,dx = e^x + C$$
$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

Trigonometric

$$\int \sin x\,dx = -\cos x + C$$
$$\int \cos x\,dx = \sin x + C$$
$$\int \sec^2 x\,dx = \tan x + C$$
$$\int \csc^2 x\,dx = -\cot x + C$$
$$\int \sec x \tan x\,dx = \sec x + C$$
$$\int \csc x \cot x\,dx = -\csc x + C$$

Inverse Trigonometric

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$$
$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$

Also: sum/difference and constant-multiple rules carry over directly to integrals.

u-Substitution — Quick Review

Idea: Reverse the chain rule. Look for a piece of the integrand whose derivative (up to a constant) also appears.

Steps:

1. Choose $u = g(x)$ — often an "inside function," something raised to a power, inside a root, in an exponent, or in a denominator.
2. Compute $du = g'(x)\,dx$, and solve for dx (or the matching piece).
3. Rewrite the *entire* integral in terms of u only — no x's should remain.
4. Integrate with respect to u.
5. Substitute $g(x)$ back in for u.

For definite integrals: either (a) convert the bounds to u-values and evaluate directly in u, or (b) back-substitute to x first, then plug in original bounds.

Example: $\int 2x\cos(x^2)\,dx$

Let $u = x^2 \Rightarrow du = 2x\,dx$.

$\int \cos(u)\,du = \sin(u) + C = \sin(x^2) + C$

Common triggers for u-sub:

- A function raised to a power, e.g. $(x^2+1)^5$
- A function inside $\sin, \cos, e^{(\cdot)}, \ln(\cdot), \sqrt{\cdot}$
- A fraction where the numerator is (close to) the derivative of the denominator