

Day 11

Exam 2 on Wed March 12th

- Arithmetic & Geometric seq / series
 - Poly. Functions
 - Rational functions
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Recall:

- hole

$$f(x) = \frac{(x+1)(x-2)}{(x+1)(x-1)}$$

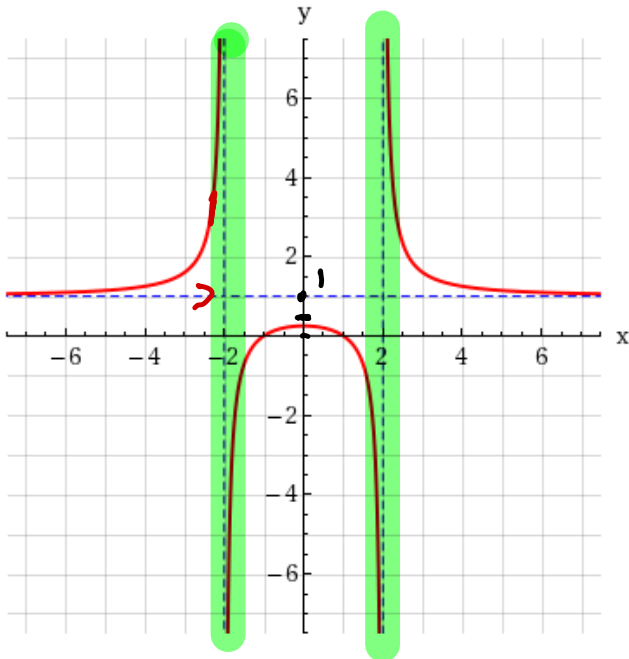
hole at $x = -1$

- V.A. at $x = 2$

- H.A. $y = 1$

4. Graphs of Rational functions

The graphs of rational functions are more complicated due to the asymptotes. Lets examine the graph of the rational function below.



• no holes

• V.A : $x=2$
 $x=-2$

• H.A : $y=1$

• x int : $x=-1$
 $x=1$

• y int : $y=\frac{1}{4}$

End behaviour

$\left. \begin{array}{l} x \rightarrow \infty \\ y \rightarrow 1 \end{array} \right\}$	$\left. \begin{array}{l} x \rightarrow -\infty \\ y \rightarrow 1 \end{array} \right\}$
$\lim_{x \rightarrow \infty} f(x) = 1$	$\lim_{x \rightarrow -\infty} f(x) = 1$

Behaviour near the V.A

$\left. \begin{array}{l} x \rightarrow 2^+ \\ y \rightarrow -\infty \end{array} \right\}$	$\left. \begin{array}{l} x \rightarrow 2^- \\ y \rightarrow -\infty \end{array} \right\}$
as x approaches the V.A $x=2$ from the right	

$\left. \begin{array}{l} x \rightarrow -2^+ \\ y \rightarrow -\infty \end{array} \right\}$	$\left. \begin{array}{l} x \rightarrow -2^- \\ y \rightarrow \infty \end{array} \right\}$
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How to graph Rational functions

Graph the rational function $f(x) = \frac{x^2 - 2x - 15}{x^2 + 4x}$.

Step 0 Factor

$$f(x) = \frac{(x-5)(x+3)}{x(x+4)}$$

Step 1 Holes? NO

Step 2 V.A. $x = 0$
 $x = -4$

Step 3 H.A. $y = 1$

Step 4 Intercepts x-int (set $y = 0$) $\frac{(x-5)(x+3)}{x(x+4)} = 0$
 $(5, 0)$ & $(-3, 0)$

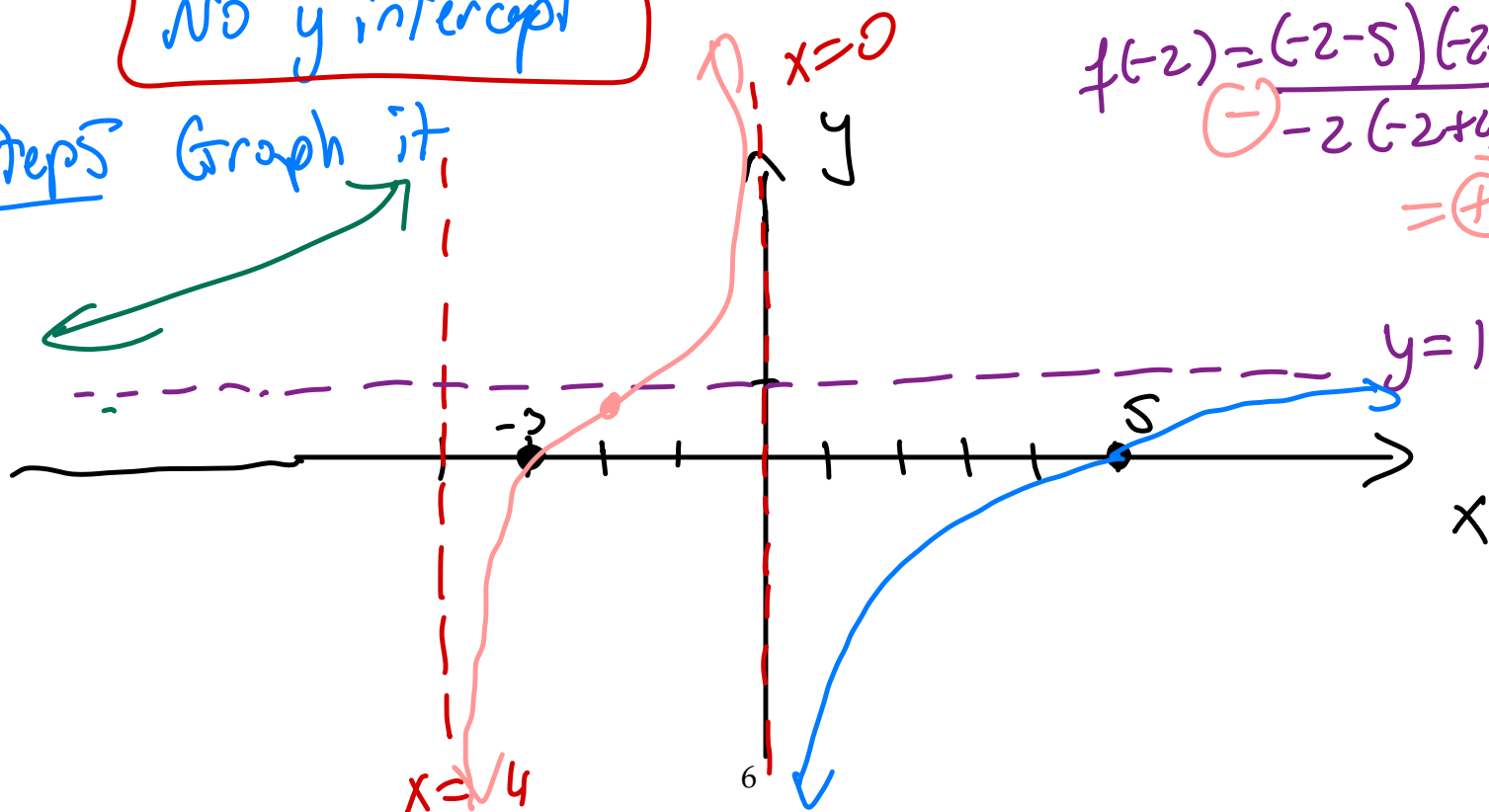
$$(x-5)(x+3) = 0$$
$$x = 5 \text{ \& } x = -3$$

$$\text{y-int : } f(0) = \frac{(0-5)(0+3)}{0(0+4)} = \text{DNE}$$

No y intercept

$$f(-2) = \frac{(-2-5)(-2+3)}{-2(-2+4)}$$
$$= \frac{(-)(-)}{(-)(+)} = (+)$$

Step 5 Graph it



Practice: Graph the rational function $f(x) = \frac{x^2 - 4x - 12}{x^2 - x - 6}$

$$0) f(x) = \frac{(x-6)(x+2)}{(x-3)(x+2)}$$

$$f(x) = \frac{x-6}{x-3}$$

1) hole is at $x = -2$

hole is a point

$$(x, y) = (-2, f(-2))$$

$$(-2, \frac{-2-6}{-2-3}) = (-2, \frac{8}{5})$$

$(-2, 1.6)$

2) V.A. $x = 3$

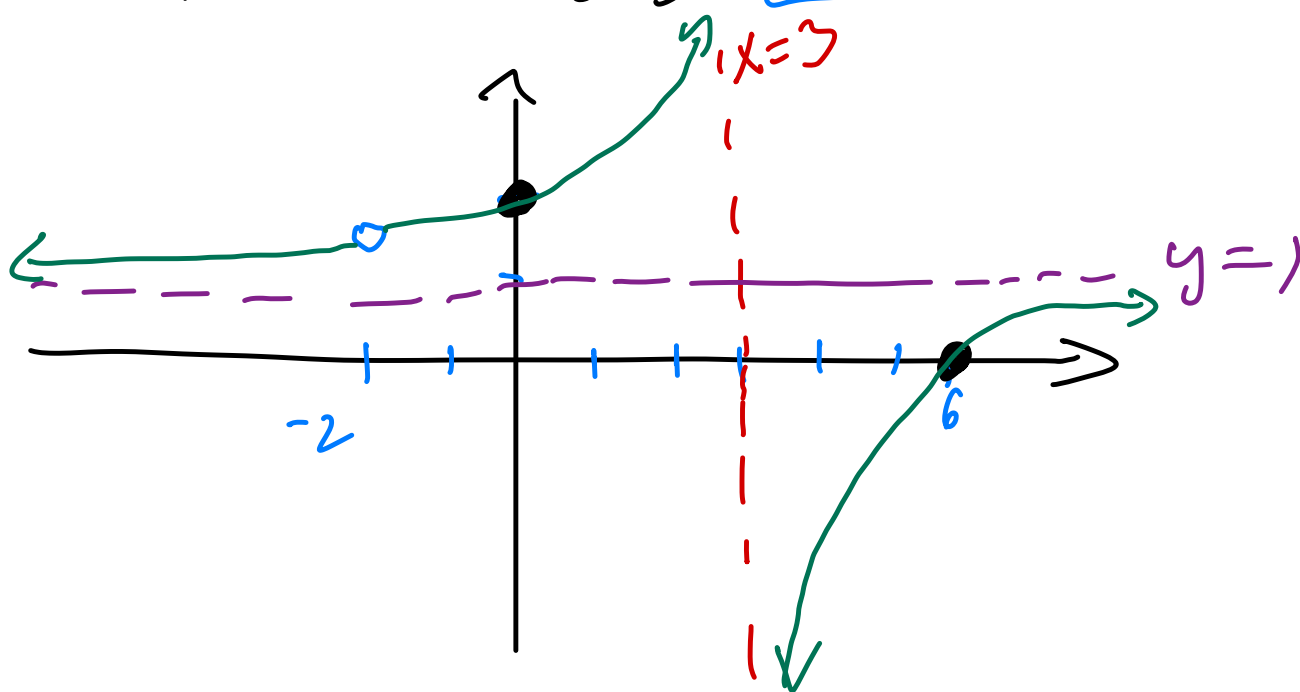
$$3) f(x) = \frac{1 \cdot x - 6}{1 \cdot x - 3}$$

$$H.A. = \frac{1}{1} = 1 \text{ so } y = 1$$

4) x intercept (numerator = 0) $x - 6 = 0$

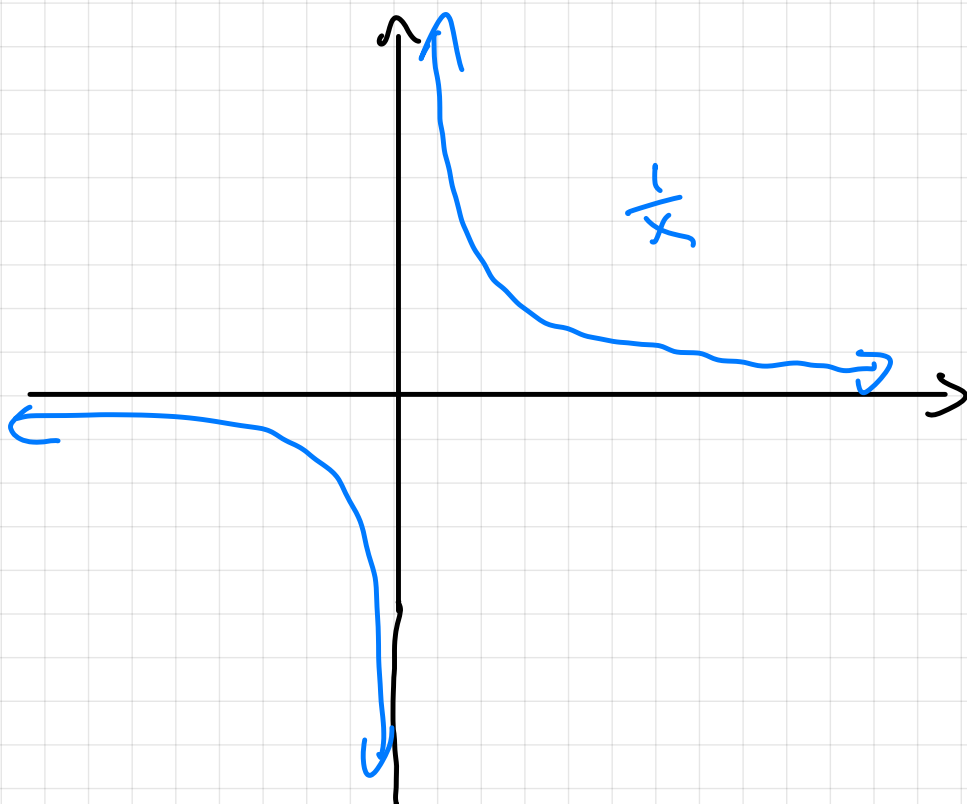
$$x = 6$$

y intercept: $f(0) = \frac{0-6}{0-3} = 2$



$$1) f(x) = \frac{x+3}{x+4}$$

parent function $\frac{1}{x}$



$$f(x) = \frac{x+3+1-1}{x+4} = \frac{\cancel{x+4} - 1}{\cancel{x+4}} = \frac{x+4}{x+4} - \frac{1}{x+4} = 1 - \frac{1}{x+4}$$

compar to $\frac{1}{x}$

$\frac{1}{x}$ add +4 $\rightarrow \frac{1}{x+4}$ left 4 units

$-\frac{1}{x+4}$ reflection over x axis

$1 - \frac{1}{x+4}$ up 1 unit.