Day 6 notes
8.2/8.3 Arithmetic and [redacted] Sequences

1. Arithmetic Sequences

In this section we are going to introduce Arithmetic Sequences. The simplest way to generate an arithmetic sequence is to start with a number a and add to it a fixed constant d, over and over again.

positive or negative

Definition: An arithmetic sequence is a sequence of the form:

$a, a+d, a+2d, a+3d, \ldots, a+(n-1)d, \ldots$

$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$

The number a is the first term and d is called the common difference.

$a_4 - a_3 = a+3d-a-2d = d$

The nth term of an arithmetic sequence is given by:

$$a_n = a + (n-1)d$$

a_1 a_2 a_3 a_4

Example 1: Is this sequence an arithmetic sequence? $13, 7, 1, -5, \cdots$.

$a_2 - a_1 = 7-13 = -6$, $a_3 - a_2 = 1-7 = -6$, $a_4 - a_3 = -5-1 = -6$

If yes, find the common difference, the next three terms, the nth term, and the 300th term of the arithmetic sequence

yes arithmetic w/ $d = -6$

$a_5 = -5 + (-6) = -11$

$a_6 = -11 + (-6) = -17$

$a_7 = -17 + (-6) = -23$

$a = 13$, $d = -6$

$a_n = a + (n-1)d = 13 + (n-1)(-6)$

$a_5 = 13 + 4(-6) = 13 - 24 = -11$

$a_{300} = a + (300-1)d = 13 + (299)(-6) = -1781$

Example 1: Write the first five terms of the sequence $a_n = -4 + 3n$. Determine whether or not the sequence is arithmetic. If it is, find the common difference.

$a_n = -4 + 3n$

$a_1 = -4 + 3(1) = -1$

$a_2 = -4 + 3(2) = 2$

$a_3 = -4 + 3(3) = 5$

$a_4 = 8$

$a_5 = 11$

$\{a_n\} = \{-1, 2, 5, 8, 11, \ldots\}$

$+3 \quad +3 \quad +3 \quad +3$

yes $d = 3$

$a = -1$

Example 2: The 11th term of an arithmetic sequence is 52, and the 19th term is 92. Find the 1000th term.

$a_{11} = 52$ $\quad a_{19} = 92$ $\quad a_{1000} = ?$

$a_n = a + (n-1)d$

$a_{11} = a + (11-1)d = 52$

$a + 10d = 52$ *

$a_{19} = a + (19-1)d = 92$

$a + 18d = 92$ *

Solve $\begin{cases} a + 10d = 52 \\ a + 18d = 92 \end{cases}$ (2) −

$a + 18d - a - 10d = 92 - 52$

$8d = 40 \Rightarrow d = 5$

Find a: $a + 10(5) = 52$

$a = 2$

$a_n = 2 + (n-1)(5)$

$a_{1000} = 2 + 999(5)$

$a_{1000} = 4997$

Arithmetic sequence recursive formula:

$a_1 = a$

$a_n = a + (n-1)d$

Recursive formula:

$a_1 = a$

$a_n = a_{n-1} + d$

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PARTIAL SUMS OF AN ARITHMETIC SEQUENCE

For the arithmetic sequence given by $a_n = a + (n-1)d$, the ***n*th partial sum**

$$S_n = a + (a+d) + (a+2d) + (a+3d) + \cdots + [a+(n-1)d]$$

is given by either of the following formulas.

1. $S_n = \frac{n}{2}[2a + (n-1)d]$
2. $S_n = n\left(\frac{a + a_n}{2}\right)$

$S_n = a_1 + a_2 + \cdots + a_n$

$S_n = n\left(\frac{a_1 + a_n}{2}\right)$ (*)

Example 4: a) Find the sum of the first 100 numbers.

$1 + 2 + 3 + 4 + \cdots + 100$

$a_1, a_2, a_3, \ldots, a_{100}$

$1, 2, 3, \ldots, 100$

$a = 1$

$d = 1$

$S_{100} = ?$

$S_{100} = 100\left(\frac{1+100}{2}\right) = 100 \cdot \frac{101}{2} = 5050$

(*) w/ $n = 100$

b) Find the sum of the first 50 odd numbers.

$1 + 3 + 5 + 7 + 9 + \cdots + 99 = ?$

$a_1, a_2, a_3, a_4, a_5, \ldots, a_{50} = 99$

$1, 3, 5, 7, 9, \ldots$

$a = 1$

$d = 2$

Find $a_{50} = ?$

$a_n = a + (n-1)d$

$a_{50} = a + (50-1)d$

$= 1 + 49 \cdot 2 \Rightarrow a_{50} = 99$

$S_{50} = 50\left(\frac{1+99}{2}\right) = 2500$

c) Sum the first 25 terms of the sequence $3, -1, -5, -9, \cdots$

$3, -1, -5, -9, \ldots$

$-4 \quad -4 \quad -4$

check if it's arithmetic

$a = 3$

$d = -4$

Want $S_{25} = ?$

$3 + (-1) + (-5) + (-9) + \cdots + (-93)$

$a_1, a_2, a_3, a_4, \ldots, a_{25}$

$a_n = a + (n-1)d$

$a_n = 3 + (n-1)(-4)$

$a_{25} = 3 + (25-1)(-4) = -93$

$S_{25} = 25\left(\frac{3 + (-93)}{2}\right) = -1125$