Math 142: Monday, Oct 28th, Day 19 Notes

Agenda for Monday, Oct 28 2024

Coming up

- Student Hours Tuesday 1-2:15pm in C162
- HW 13 due today
- Excel Assignment 3 due Wednesday

Recall:

Discrete Random Variables

X	P(X)

L_1 L_2

all possible

Use 1 Var Stats (L_1, L_2)

to get

mean $= \mu = E(x)$

st. dev

variance

Binomial Random Variables

satisfy 4 conditions

$X \sim B(n, p)$

# of trials

pros of success

binomialpdf$(n, p, k) = P(X = k)$

binomialcdf$(n, p, k) = P(x \leq k)$

cumulative

Mean, variance, st dev

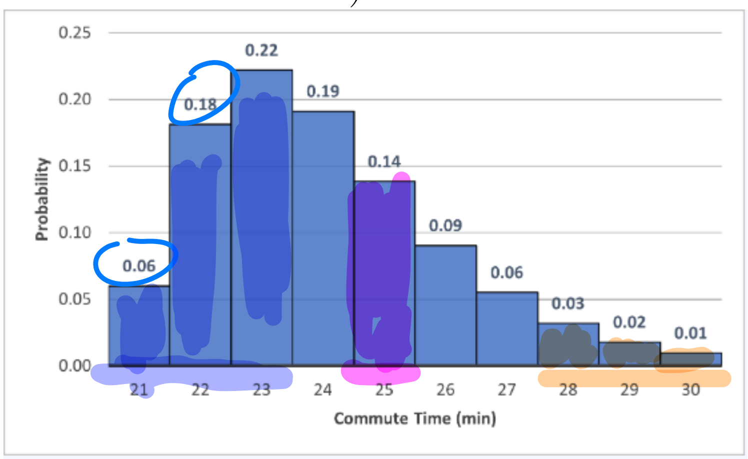
$\mu =$ mean $= np$ $\qquad q = 1 - p$

$\sigma^2 =$ variance $= npq$

$\sigma =$ st. dev $= \sqrt{npq}$

So far we discussed discrete probability distributions which summarized the probabilities of experiments with a finite number of possible outcomes (for discrete random variables). However, in some situations, it makes sense to consider the outcomes of an experiment that fall within a continuous range of outcomes, in which case there are infinitely many possible outcomes (for continuous random variables).

Example 1. Consider the following probability distribution for the amount of time you spend commuting to work on a given day (rounded to the nearest number of minutes).



x	$P(x)$
21	
22	
⋮	
30	

1. What is the probability that your commute takes 25 minutes?
2. What is the probability that your commute takes 23 minutes or less?
3. What is the probability that your commute takes longer than 27 minutes?

1) $P(x=25) = 0.14$

2) $P(x \le 23) = P(x=21)+P(x=22)+P(x=23) = 0.06+0.18+0.22$
$= 0.46$

3) $P(x>27) = P(x=28)+P(x=29)+P(x=30)$
$= 0.03+0.02+0.01$
$= 0.06$

A given commute might take closer to 23.25 minutes than 23 minutes, a level of precision that we ignored. In theory, your commute could take any number of minutes, including decimal numbers of minutes like 21.326 or even 26.202002000200002.... Although we would never be able to measure your travel time with an infinite amount of precision, it makes sense to say that your actual travel time could be any of the infinitely-many real numbers within a reasonable range; all of these numbers are possible outcomes.

Consider what happens to the commute time probability distribution if we start splitting outcomes into more precise categories, from the nearest minute, to the nearest half-minute, to the nearest quarter-minute, and so on:

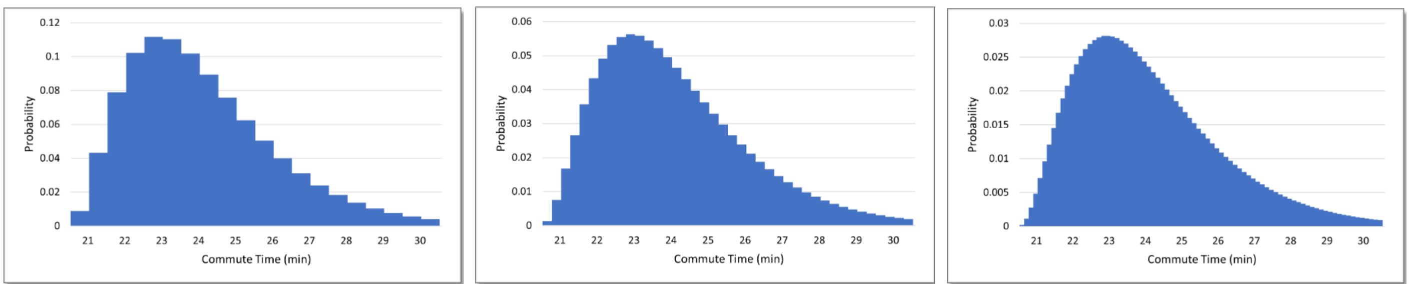
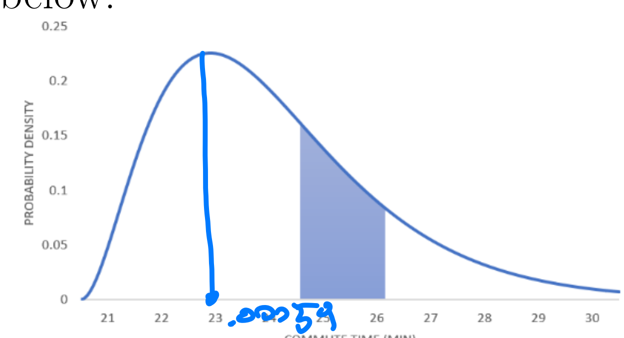


Figure 1: (a) nearest half-minute (b) nearest quarter-minute (c) nearest eighth-minute

If we continue this process of splitting the distribution into more and more precise outcomes, these graphs will approach a smooth curve, as in Figure below.



This curve is called the **probability density function**, or the **continuous probability distribution** for the experiment. The probability density function will tell us the probability corresponding to any interval.

Probability Density Functions

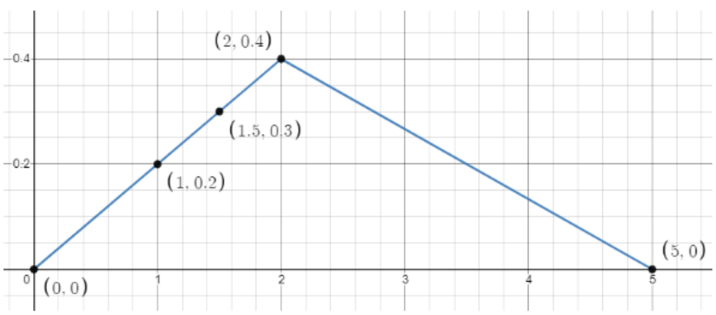
$P(23 < x < 23.12)$

- A probability density function is a curve that fully describes the probabilities involved in an experiment where the collection of outcomes is a continuous interval.

$P(x = 23.00597254) = ? = 0$

- The probability of any given outcome of such an experiment is typically 0, so we only measure the probability that the result of the experiment falls within an interval of outcomes.
- Given an interval of outcomes, the probability associated to the interval is the area under the probability density function over that interval.
- The total area under a probability density function is 1, since this is the probability that any outcome in the interval of all possible outcomes occurs.

Example 2. Suppose a computer program outputs a random number between 0 and 5 with the following probability density function.



next class

1. What is the probability that the output is greater than 2?
2. What is the probability that the output is between 1 and 2?
3. Practice on your own: What is the probability that the output is at least 1.5? [1]

[1]answer for c) is 0.775

Example 3. Suppose the distribution of arrival times is **uniform** with the minimum of 2 minutes and maximum of 15 minutes for Fire trucks reaching the scene after being called.

1. What is the probability that a fire truck will reach the scene within 7 to 10 minutes?
2. What is the probability that a fire truck will reach fire less than 6.5 minutes?

next class

Notation and Properties of the Uniform Distribution

- When a continuous random variable, X, is uniformly distributed we write
- The mean of a uniform probability distribution is
- Variance of a uniform probability distribution
- The standard deviation
- Probability = Area = (Base)(height)

Group Activity

Assume you take a test consisting of 15 multiple choice question, 4 choices for each question and you randomly guess on each question with no pattern. This is a binomial experiment with $n = 15$ trials and probability of success $p = 0.25$.

a) Find probability of at most 3 questions correct:

b) Find the probability of less than 6 questions correct:

c) Find the probability of at least 9 questions correct:

d) Find the probability of greater than 3 questions correct:

e) Mean of this binomial distribution:

f) Variance of this binomial distribution:

g) Standard deviation of this binomial distribution:

a) $P(x \le 3) = \text{binomialcdf}(15, 0.25, 3)$

b) $P(x < 6) = \text{binomialcdf}(15, 0.25, 5)$

c) $P(x \geq 9) = 1 - \text{binomialcdf}(15, 0.25, 8)$

d) $P(x > 3) = 1 - \text{binomialcdf}(15, 0.25, 2)$

e) $\mu = np = (15)(0.25)$

f) $\sigma^2 = npq = (15)(0.25)(0.75)$

g) $\sigma = \sqrt{(15)(0.25)(0.75}$