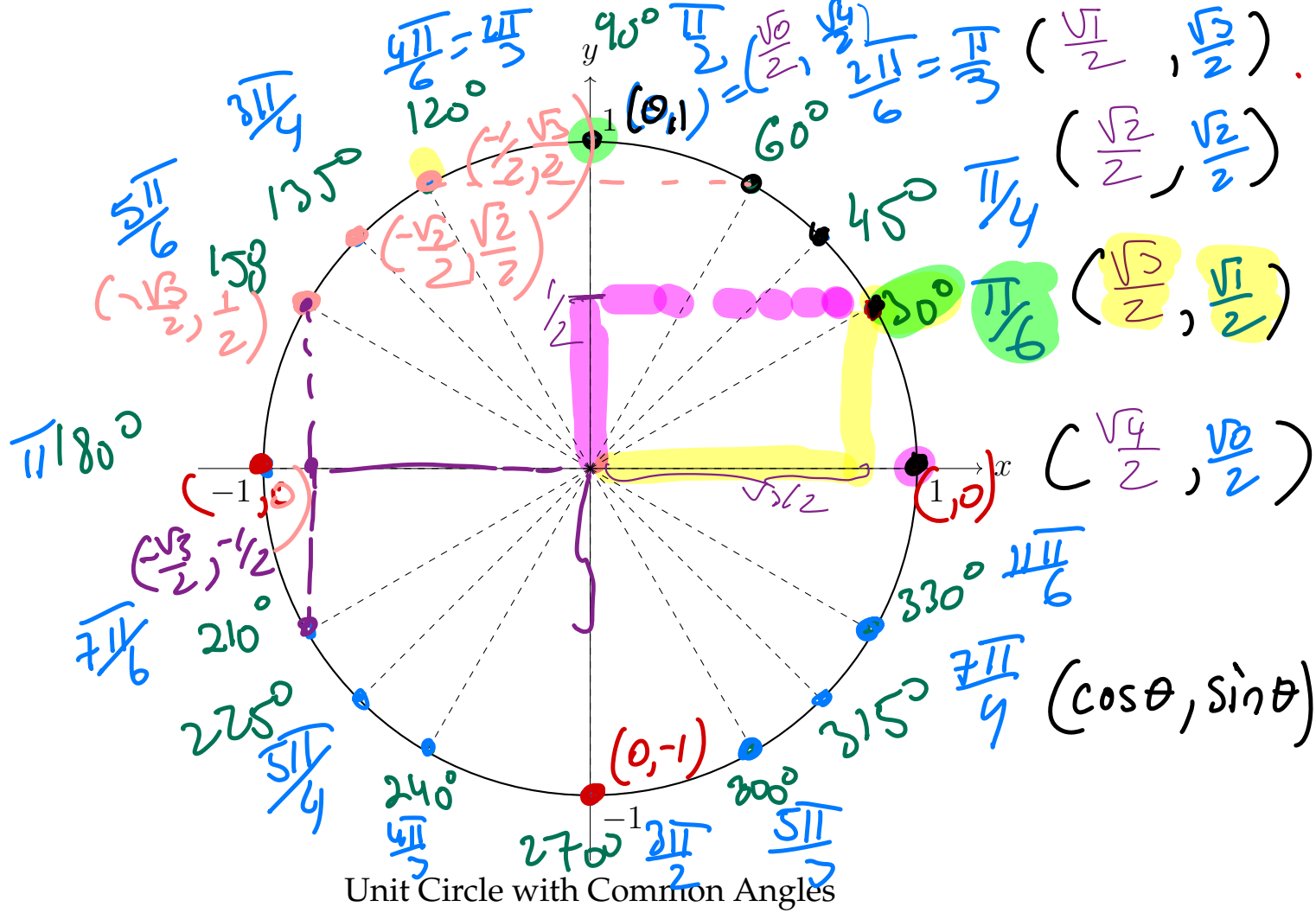




5.3 Evaluating Trigonometric Functions

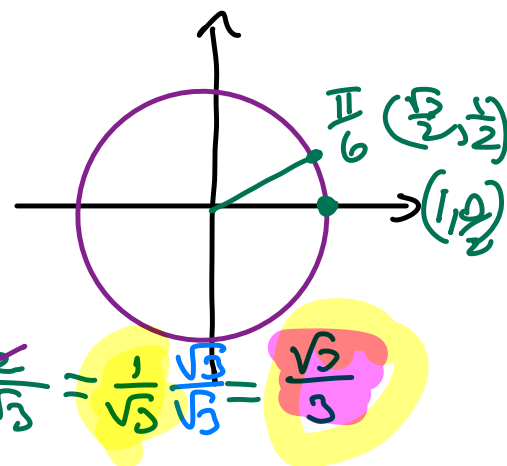
Example 7: Evaluating Trig Functions for Special Angles

Find the values of all six trigonometric functions for $\theta = 30^\circ$.

Solution:

For $\theta = 30^\circ = \frac{\pi}{6}$ radians, the point on the unit circle is:

$$P(\cos 30^\circ, \sin 30^\circ) = P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$



Therefore:

$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\csc 30^\circ = \frac{1}{\sin 30^\circ} = \frac{1}{1/2} = 2$$

$$\sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cot 30^\circ = \frac{1}{\tan 30^\circ} = \sqrt{3}$$

Example 8: Finding Trig Functions from a Point

If $P(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ is a point on the unit circle corresponding to angle θ , find all six trigonometric functions of θ .

Solution:

Since the point is on the unit circle, we know that.



$$\cos \theta = -\frac{\sqrt{2}}{2}$$
$$\sin \theta = \frac{\sqrt{2}}{2}$$

The other trigonometric functions can be found using these values:

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{-1} = -1$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1$$
$$\csc \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$
$$\sec \theta = -\sqrt{2}$$
$$\cot \theta = -1$$

5.4 Domains of Trigonometric Functions

Definition : Domains of Trigonometric Functions

The domains of the six trigonometric functions are:

$\sin \theta$: All real numbers

$\cos \theta$: All real numbers

$\tan \theta$: All real numbers except $\theta = \frac{\pi}{2} + n\pi$ (where n is an integer)

$\csc \theta$: All real numbers except $\theta = n\pi$ (where n is an integer)

$\sec \theta$: All real numbers except $\theta = \frac{\pi}{2} + n\pi$ (where n is an integer)

$\cot \theta$: All real numbers except $\theta = n\pi$ (where n is an integer)

odd multiples of $\frac{\pi}{2}$: $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots$
 $\pm\pi, \pm2\pi, \pm3\pi, \dots$
(multiples of π)

5.5 Even and Odd Trigonometric Functions

Definition : Even and Odd Functions

A function f is **even** if $f(-x) = f(x)$ for all x in the domain. A function f is **odd** if $f(-x) = -f(x)$ for all x in the domain.

Note : Even and Odd Trigonometric Functions

- $\cos \theta$ & $\sec \theta$ are even functions.
- $\sin \theta$, $\csc \theta$, $\tan \theta$, $\cot \theta$ are odd functions

5.6 Periodicity of Trigonometric Functions

Definition : Periodic Functions

A function f is **periodic** if there exists a positive real number p such that $f(x+p) = f(x)$ for all x in the domain. The smallest such value of p is called the **period** of the function.

Note : Periods of Trigonometric Functions

- $\sin \theta$, $\csc \theta$, $\cos \theta$, $\sec \theta$ have a period of 2π (values start repeating after 2π)
- $\tan \theta$, $\cot \theta$ have a period of π

6 Summary of Key Concepts

1. **Angles** are formed by two rays with a common vertex; they have an initial side and a terminal side.
2. **Standard position** means the angle's vertex is at the origin and the initial side is along the positive x-axis.
3. **Positive angles** rotate counterclockwise; **negative angles** rotate clockwise.
4. **Coterminal angles** share the same terminal side and differ by multiples of 360° or 2π radians.
5. **Radians** and **degrees** are two common angle measures: π radians $= 180^\circ$.
6. **Complementary angles** sum to 90° or $\frac{\pi}{2}$ radians.
7. **Supplementary angles** sum to 180° or π radians.
8. **Arc length** formula: $s = r\theta$ (where θ is in radians)
9. **Area of a sector** formula: $A = \frac{1}{2}r^2\theta$ (where θ is in radians)
10. **Unit circle** has radius 1 and is centered at the origin. Points on the unit circle are $(x, y) = (\cos \theta, \sin \theta)$.
11. **Six trigonometric functions**: sine, cosine, tangent, cosecant, secant, and cotangent.
12. **Even functions**: $\cos \theta$ and $\sec \theta$
13. **Odd functions**: $\sin \theta$, $\csc \theta$, $\tan \theta$, and $\cot \theta$
14. **Periodicity**: $\sin \theta$, $\cos \theta$, $\csc \theta$, and $\sec \theta$ have period 2π ; $\tan \theta$ and $\cot \theta$ have period π .