Probability Section 3.5 Counting

Combinations and Permutations

In English we use the word "combination" loosely. We say things like:

- "My fruit salad is a combination of apples, grapes and bananas"

ABD = ABC order does not matter

- "The combination to the safe is 472".

order matter

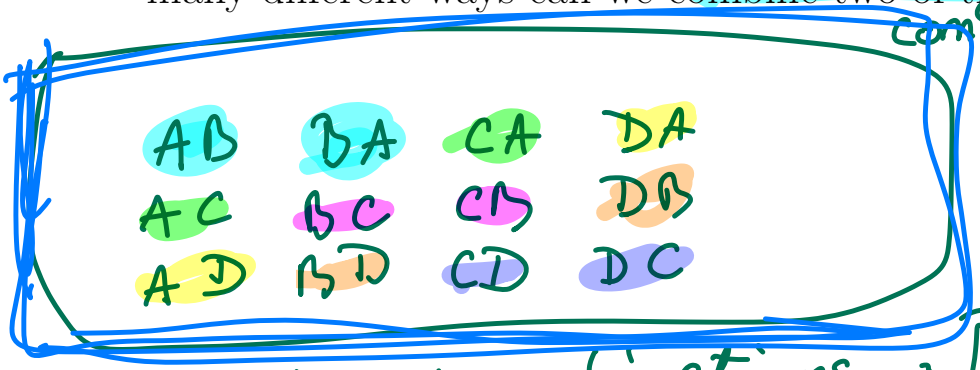
427 ≠ 472

When the order doesn't matter, it is a **Combination**.
When the order does matter it is a **Permutation**.

Permutations and Combinations: Consider having the letters A, B, C, D. Let's choose two of the letters.
In how many different ways can we arrange two of the 4 letters? In how many different ways can we combine two of the 4 letters?

permutation

combination



- number of combinations : 6

- number of permutations : 12

Permutations: We use permutations when n is the number of things to choose from, and we choose r of them, no repetitions, order matters.

Formula:

$$nPr = \frac{n!}{(n-r)!}$$

$n = 4$ letters, $r = 2$, no repetitions, order matters, $AB \neq BA$

$$4P2 = \frac{4!}{(4-2)!} = \frac{1 \cdot 2 \cdot 3 \cdot 4}{1 \cdot 2} = 12$$

Calculator instructions:

MATH → PROB → nPr → 4 → 2 → ENTER → 12

Example 1. A volleyball team has 10 players, how many 6 player starting line up are possible?

$n = 10$ players ✓
$r = 6$ players ✓
no repetition ✓
order matters ✓

$$nPr = 10P6 = \frac{n!}{(n-r)!} = \frac{10!}{(10-6)!}$$

$$= \frac{10!}{4!} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10}{4!} = 2100$$

$$= 151{,}200 \text{ ways}$$

$$10P6 = 151{,}200$$

n = 4 letters

Combinations We use combination when n is the number of things to choose from, and we choose r of them, no repetition, order doesn't matter.

r = 2

Formula:

$$nCr = \frac{n!}{(n-r)!\,r!}$$

$$4C2 = \frac{4!}{(4-2)!\,2!}$$

$$= \frac{4!}{2!\cdot 2!} = \frac{1\cdot 2\cdot 3\cdot 4}{1\cdot 2\cdot 1\cdot 2}$$

= 6 ways

Calculator instructions:

MATH → PROB → nCr

r = 4

Example 2. Four lawyers are to be selected from a group of 30 lawyers to work on a project.

n = 30

1. How many different ways can the lawyers be selected to work on the project?

order does not matter A, B, C, D is same D, C, B, A.

nCr = 30C4 = 27,405.

2. In how many ways can the group of 4 be selected if a certain lawyer must work on the project?

Pick the one lawyer

29C3 = 3654

Example 3. In a student club with 9 sophomores and 11 freshman members, a 5- member committee will be randomly chosen. How many ways can a committee be chosen with 3 sophomores and 2 freshmen?

(9C3) · (11C2) = (84)(55)

soph

= 4620 ways