Math 142: Monday, Oct 7th, Day 15 Notes

Agenda for Monday, Oct 7th 2024

Coming up

- ! **HW 10** due Wednesday, Oct 9th
- Group Quiz 4 Wednesday on Probability
- ! Exam 2 on Wednesday Oct 16th
- Student Hours Tuesday, 1-2:15pm in C162
- Carrera de los Lancers

Recall:

Example 1. Two psychologists surveyed 478 children in elementary schools in Michigan. Among other questions, they asked the students whether their primary goal was to get good grades, to be popular, or to be good at sports.

	Grades	Popular	Sports	Total
Boy	117	50	60	227
Girl	130	91	30	251
Total	247	141	90	478

1. If we select a student at random from this study, what is the probability we select a girl ?

$\ast$ P(girl)= $\frac{251}{478} = 0.525$

2. What's the probability of selecting a girl whose goal is to be popular?

$\ast$ P(girl and Sports)= $\frac{30}{478} = 0.063$ $P(\text{girls and popular}) = \frac{91}{478}$

3. What's the probability of selecting any student whose goal is to excel at sports?

P(sports)= $\frac{90}{478} = 0.188$

4. What if we are **given** the information that the selected student is a girl? Would that change the probability that the selected student's goal is sports?

$P(\text{sports given girl}) = \frac{30}{251}$

$P(\text{sports} \mid \text{girl}) = \frac{30}{251} = 0.120$

$P(\text{sports} \mid \boxed{\text{girl}}) = \dfrac{\frac{30}{478}^{\ast}}{\frac{251}{478}\ast} = \dfrac{P(\text{girl and sports})}{P(\text{girl})}$

$$\boxed{P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}}$$

Conditional probability

$P(* \mid ☆) = \frac{P(* \text{ and } ☆)}{P(☆)}$

A probability that takes into account a given condition such as this is called a **conditional probability**.

To find the probability of the event A happening giving that event B occured we use the formula:

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$P(B \mid A) = \frac{P(B \text{ and } A)}{P(A)}$$

Example 2. Our survey found that 33% of college students are in a relationship, 25% are involved in sports, and 11% are both.

R Sp

$P(R \text{ and } Sp) \overset{?}{=} P(R) \cdot P(Sp) = (.33)(.25)$

$.11 \neq$

$P(R) = .33$

$P(Sp) = .25$

$\boxed{P(R \text{ and } Sp) = .11}$

1. While dining in a campus facility open only to students, you meet a student who is on a sports team. What is the probability that your new acquaintance is in a relationship?

$$P(R \mid Sp) = \frac{P(R \text{ and } Sp)}{P(Sp)} = \frac{.11}{.25} = .44$$

2. While dining in a campus facility open only to students, you meet a student who is in a relationship. What is the probability that your new acquaintance is involved in sports?

$$P(Sp \mid R) = \frac{P(R \text{ and } Sp)}{P(R)} = \frac{.11}{.33} = .333$$

Caution: $P(R \mid Sp) \neq P(Sp \mid R)$

Independent events revisited:

1. Consider drawing two cards (one at a time) from a deck of cards **with replacement**. Let K_{1st} be the event of "drawing a King the first time" and K_{2nd} be the event of "drawing a King the second time". Find:

$P(K_{2nd}) =$ 4/52

indep

$$\boxed{P(K_{2nd}|K_{1st})} = \frac{P(K_{2nd} \text{ and } K_{1st})}{P(K_{1st})} = \frac{P(K_{2nd}) \cdot \cancel{P(K_{1st})}}{\cancel{P(K_{1st})}} = P(K_{2nd})$$

$P(A|B) = P(A)$

Event A and B are independent if and only if:

- $P(A \text{ and } B) = P(A) \cdot P(B)$
- $P(A|B) = P(A)$
- $P(B|A) = P(B)$

use any of these to decide if events are independent

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2. Consider drawing two cards (one at a time) from a deck **without replacement**. Let K_{1st} be the event of "drawing a King the first time" and K_{2nd} be the event of "drawing a King the second time". Find:

can't break it down into $P(K_{2nd}) \cdot P(K_{1st})$

$$P(K_{2nd}|K_{1st}) = \frac{3}{51} = \frac{P(K_{2nd} \text{ and } K_{1st})}{P(K_{1st})}$$

$$P(K_{2nd}|K_{1st}) = \frac{P(K_{2nd} \text{ and } K_{1st})}{P(K_{1st})}$$

$$\boxed{P(K_{2nd} \text{ and } K_{1st}) = P(K_{1st}) \cdot P(K_{2nd}|K_{1st})}$$

General Multiplication Rule

The probability that two events A and B will occur in sequence is

$$P(A \text{ and } B) = P(A) \cdot P(B|A).$$

If events A and B are independent, then the rule can be simplified to

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

because $P(B|A) = P(B)$ when A, B independent

Example 3. In a survey, 510 adults were asked if they drive a pickup truck and if they drive a Ford. The results showed that one in six adults surveyed drives a pickup truck, and three in ten adults surveyed drive a Ford. Of the adults surveyed that drive Fords, two in nine drive a pickup truck.

1. Find the probability that a randomly selected adult drives a pickup truck, **given** that the adult drives a Ford.

   $P(\text{Truck} \mid \text{Ford}) = 2/9$

2. Are the events "driving a Ford" and "driving a pickup truck" independent or dependent? Explain.

   Check one of these:
   - $P(\text{Ford} \mid \text{Truck}) \overset{?}{=} P(\text{Ford})$
   - $P(\text{Truck} \mid \text{Ford}) \overset{?}{=} P(\text{Truck})$: $\frac{2}{9} \neq \frac{1}{6}$ so not indep.
   - $P(\text{Truck and Ford}) \overset{?}{=} P(\text{Truck})\, P(\text{Ford})$

3. Find the probability that a randomly selected adult drives a Ford pick up truck.

   $P(\text{Truck and Ford}) = P(\text{Ford}) \cdot P(\text{Truck} \mid \text{Ford})$
   not indep.
   $= \frac{3}{10} \cdot \frac{2}{9} = \frac{1}{15}$

next class

Using Tree Diagrams

The kind of picture that helps us look at conditional probabilities and multiplication rule for dependent events is called a **tree diagram**.

Example 4. According to a study 44% of college students engage in binge drinking, 37% drink moderately, and 19% abstain entirely. Another study finds that among binge drinkers, 17% have been involved in an alcohol-related automobile accident, while among non-bingers of the same age, only 9% have been involved in such accidents.

1. Build a tree diagram with corresponding probabilities on each possible outcome.

2. What's the probability that a randomly selected college student will be a binge drinker **and** has had an alcohol-related car accident?

3. What's the probability that a randomly selected college student will have an alcohol-related car accident **given** they are a moderate drinker?

4. What's the probability that a randomly selected college student will be a moderate drinker **and** has had no alcohol-related car accident?