# Math 140: Chapter 3 Probability

## 3.1 Probability Basic Concepts

Experiment: is an action where the results are random

**Example 1.** Examples of experiments:

- tossing a coin $S = \{H, T\}$ $n(S) = 2$
- rolling a die $S = \{1, 2, 3, 4, 5, 6\}$ $n(S) = 6$

Outcome: is a single result from performing the experiment once.

Sample Space: is a list of all possible outcomes

Notation for sample space: is denoted with S

$S = \{$ all possible outcomes $\}$

Size of sample space: is denoted with $n(S)$

**Example 2.** What is the sample space for Rolling a 6-sided die and flipping a coin?

$S =$ {1T, 2T, 3T, 4T, 5T, 6T, 1H, 2H, 3H, 4H, 5H, 6H}

$n(S) =$ 12

Event is a subset (smaller list) of the sample space.

**Example 3.** Rolling a number less than three and tails

$E =$ {1T, 2T} $n(E) = 2$

size of events

There are two basic ways to calculate probabilities: theoretical probability and empirical probability.

Classical (Theoretical) Probability is used when each outcome is equally likely.

$$P(E) = \frac{n(E)}{n(S)}$$

$n(E)$: size of event E (or number of favorable outcomes to event E).

$P(E)$: probability of event E

$n(S)$: size of sample space.

ex 3: $P(E) = \frac{n(E)}{n(S)} = \frac{2}{12} = \frac{1}{6}$

Rounding Rule for probability: use 3 significant digits

A probability is given as a fraction, decimal or percentage

$P(E) = \frac{1}{6} = 0.1666\overline{6} = 0.167$

ex: $P(E) = 0.0000\,30666$

$P(E) = 0.0000307$

3 significant digits

**Example 4.** Classical probability Experiment: Rolling a fair 12 sided die

S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}  n(S) = 12

Event A: rolling a prime number  A = {2, 3, 5, 7, 11}, n(A) = 5

Event B: rolling a number divisible by 5  B = {5, 10}  n(B) = 2

Event C: rolling a number greater than 4 which is divisible by 3

C = {6, 9, 12}  n(C) = 3

Find the following probabilities.

$P(A) = \frac{n(A)}{n(S)} = \frac{5}{12}$

$P(B) = \frac{2}{12} = \frac{1}{6}$

$P(C) = \frac{3}{12} = \frac{1}{4}$

Empirical (Experimental) Probability  used when events are not equally likely (or it's the long term relative frequency).

$P(A) = \frac{\text{number of times A happens}}{\text{number of total trials}} = \frac{\text{frequency}}{\text{sample size.}}$

**Example 5.** Julia made a die for a game. It had 5 faces, labeled 1-5. Because of the shape of the die, it was biased in favor of rolling a 3. She experimented to estimate how likely it was for a 3 to be rolled. Julia rolled the die 100 times, and the number of times she rolled a 3 was 45.

$P(3) = \frac{45}{100} = 0.45$

**Example 6.** Empirical Probability: Choosing a student from a survey

| | Accounting | Non Accounting | |
|---|---|---|---|
| Freshman | 20 | 50 | 70 |
| Soph | 14 | 56 | 70 |
| | 34 | 106 | 140 |

1. What is the probability of choosing a student who is majoring in accounting?

$$P(\text{Accounting}) = \frac{34}{140} = \frac{\text{frequency}}{\text{sample size}} = 0.2428 = 0.243$$

2. What is the probability of choosing a student who is a sophmore?

$$P(\text{Soph}) = \frac{70}{140} = \frac{1}{2}$$

3. What is the probability of choosing a student who is not majoring in accounting?

$$P(\text{not Accounting}) = \frac{106}{140} = 0.757$$

impossible $P(7) = 0$ certainty

**Properties of Probability**

1. For any event $A$, $\mathbf{0 \le P(A) \le 1}$. That is, the probability of any event is always a number between 0 and 1.
2. The probability of any event $A$ is equal to the sum of the probabilities of the individual outcomes in $A$.
3. The sum of the probabilities of all outcomes in $S$ must equal 1. This is true whether $S$ consists of equally likely outcomes.

② Rolling a six sided die

event A is even number

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 4, 6\}$

$$P(A) = P(2) + P(4) + P(6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

③ $P(S) = 1$