

Fundamental Trigonometric Identities

Reciprocal Identities

$$\text{R1: } \csc(x) = \frac{1}{\sin(x)}$$

$$\text{R2: } \sec(x) = \frac{1}{\cos(x)}$$

$$\text{R3: } \cot(x) = \frac{1}{\tan(x)}$$

$$\text{R4: } \sin(x) = \frac{1}{\csc(x)}$$

$$\text{R5: } \cos(x) = \frac{1}{\sec(x)}$$

$$\text{R6: } \tan(x) = \frac{1}{\cot(x)}$$

Quotient Identities

$$\text{Q1: } \tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\text{Q2: } \cot(x) = \frac{\cos(x)}{\sin(x)}$$

Pythagorean Identities

$$\text{P1: } \sin^2 x + \cos^2 x = 1$$

$$\text{P2: } 1 + \tan^2 x = \sec^2 x$$

$$\text{P3: } 1 + \cot^2 x = \csc^2 x$$

Even-Odd Identities

$$\text{C1: } \sin(-x) = -\sin x$$

$$\text{C2: } \cos(-x) = \cos x$$

$$\text{C3: } \tan(-x) = -\tan x$$

$$\text{C4: } \csc(-x) = -\csc x$$

$$\text{C5: } \sec(-x) = \sec x$$

$$\text{C6: } \cot(-x) = -\cot x$$

Cofunction Identities

$$\text{CF1: } \sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\text{CF2: } \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\text{CF3: } \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\text{CF4: } \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\text{CF5: } \sec\left(\frac{\pi}{2} - x\right) = \csc x$$

$$\text{CF6: } \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

Addition and Subtraction Identities

$$\text{A1: } \sin(A + B) = \sin A \cdot \cos B + \sin B \cdot \cos A$$

$$\text{S1: } \sin(A - B) = \sin A \cdot \cos B - \sin B \cdot \cos A$$

$$\text{A2: } \cos(A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$\text{S2: } \cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$

$$\text{A3: } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} \quad \text{S3: } \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$$

Double Angle Identities

$$\text{D1: } \sin(2\theta) = 2 \sin \theta \cdot \cos \theta$$

$$\text{D2a: } \cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$\text{D2b: } \cos(2\theta) = 1 - 2 \sin^2 \theta$$

$$\text{D2c: } \cos(2\theta) = 2 \cos^2 \theta - 1$$

$$\text{D3: } \tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Half-Angle Identities

$$\text{H1: } \sin\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 - \cos A}{2}} \quad \text{H2: } \cos\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\text{H3a: } \tan\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}} \quad \text{H3b: } \tan\left(\frac{A}{2}\right) = \frac{1 - \cos A}{\sin A}$$

$$\text{H3c: } \tan\left(\frac{A}{2}\right) = \frac{\sin A}{1 + \cos A}$$

$$\text{OR: } \sin^2 A = \frac{1}{2}(1 - \cos 2A) \quad \text{OR: } \cos^2 A = \frac{1}{2}(1 + \cos 2A)$$