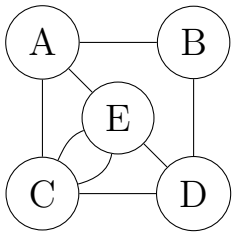
MTH 140: Practice Exam 1 Key

College of Lake County

1. Use the following graph to answer the questions:



(a) The degree of vertex A is 3.

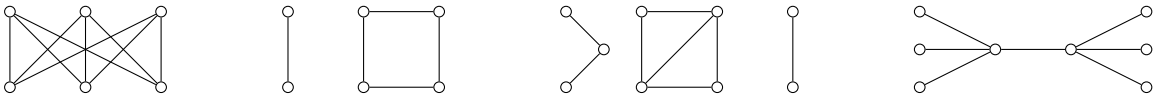
(b) The degree of vertex C is 4.

(c) Is the graph connected? yes. There is a path from any two vertices.

(d) Does the graph have a Euler circuit? NO

2. Use the figure below to answer the question.

Graph 1 Graph 2 Graph 3 Graph 4



In which graph is it true that every edge is a bridge?

a. Graph 1

b. Graph 2

c. Graph 3

d. Graph 4

e. None of the above

3. Suppose that a graph has 4 vertices each of degree 2. How many edges does the graph have?

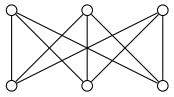
a. 6 edges

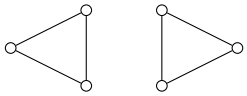
b. 16 edges

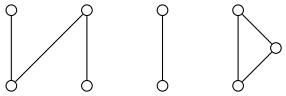
c. 8 edges

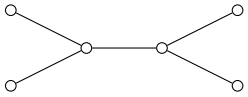
d. 4 edges

4. Which graph has an Euler circuit?

Graph 1 

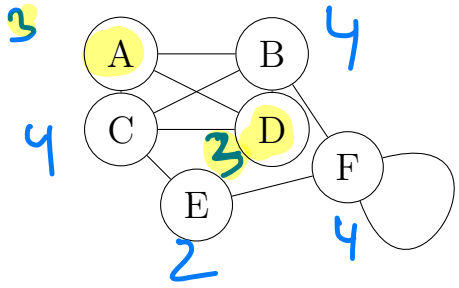
Graph 2 

Graph 3 

Graph 4 

a. Graph 1
b. Graph 2
c. Graph 3
d. Graph 4
e. None of the above

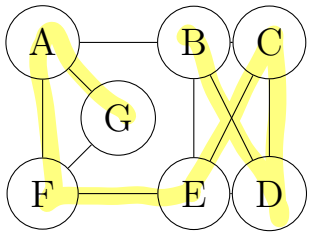
5. Given the graph, which of the following statements is true?



a. The graph has an Euler path from vertex A to vertex D.
b. The graph has an Euler circuit.
c. The graph has both an Euler circuit and an Euler path of the above are true.
d. The graph has neither an Euler circuit nor an Euler path
e. none of the above

only two odd vertices

6. Consider the graph below.



(a) Is the graph a network?

yes

(b) Does the graph have any bridges?

NO

(c) Give an example of two adjacent vertices.

A & F

(d) Give an example of two adjacent edges.

AF, FG

(e) Does the graph have a Hamilton circuit. If, yes list one Hamiltonian circuit.

yes AGFECDBA

7. How many edges are in the complete graph K_7?

$\frac{7 \cdot 6}{2} = 7 \cdot 3 = 21$

K_N has $\frac{N(N-1)}{2}$ edges

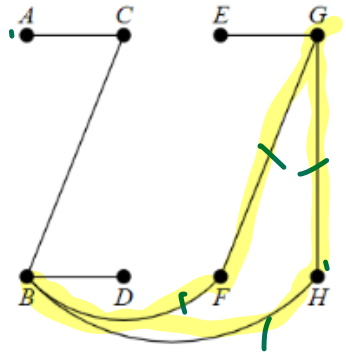
8. How many Hamilton circuits are in the complete graph K_7?

6!

K_N has $(N-1)!$ HC

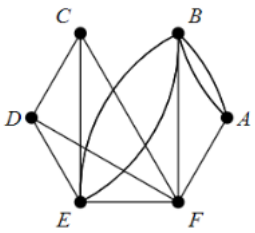
9. How many spanning trees does this graph have?

8 vertices
7 edges



4 sp. trees to correspond to each edge from GFBHG circuit

10. Consider the following graph.

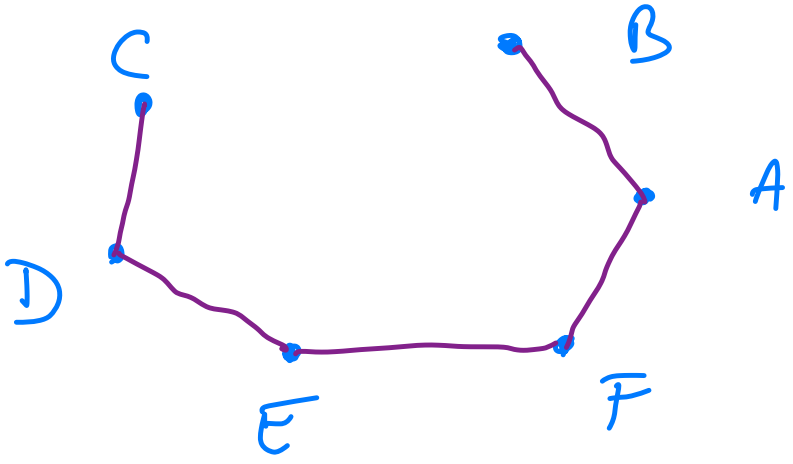


6 vertices → sp. tree has 5 edges
11 edges

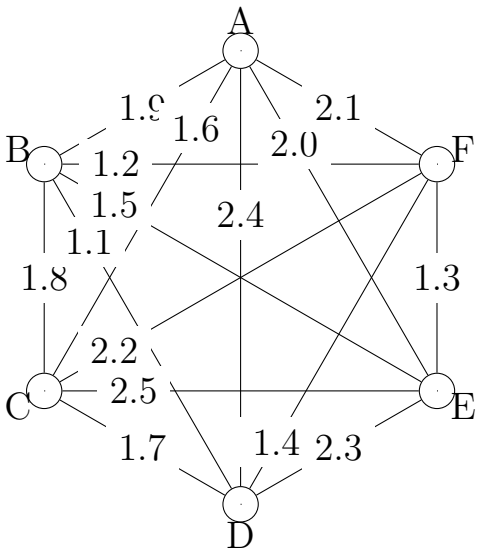
(a) How many edges should a tree corresponding to this graph have?

5 edges

(b) Draw one spanning tree corresponding to this graph.



11. A delivery truck must deliver packages to 6 different store locations (A, B, C, D, E, and F). The trip must start and end at C. The graph below shows the distances (in miles) between locations:



We want to minimize the total distance traveled. The nearest-neighbor algorithm applied to the graph yields the following solution:

a. C, A, E, F, B, D, C.

b. C, A, B, D, F, E, C.

c. C, B, D, F, E, A, C.

d. C, D, E, F, A, B, C.

e. None of the above

12. A traveling salesman's territory consists of the 5 cities shown on the following mileage chart.

	Boston	Buffalo	Chicago	Columbus	Louisville
Boston	0	446	963	735	941
Buffalo	446	0	522	326	532
Chicago	963	522	0	308	292
Columbus	735	326	308	0	209
Louisville	941	532	292	209	0

The salesman must organize a round trip that starts and ends at Louisville (his hometown) and will pass through each of the other four cities exactly once, and of course wishes to do it in the most economical way possible.

The Nearest Neighbour algorithm applied to this problem yields the following solution:

a. Louisville, Columbus, Chicago, Buffalo, Boston, Louisville.

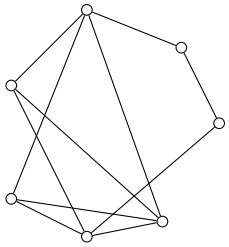
b. Louisville, Columbus, Buffalo, Boston, Chicago, Louisville.

c. Louisville, Chicago, Buffalo, Boston, Columbus, Louisville.

d. Louisville, Boston, Buffalo, Chicago, Columbus, Louisville.

e. None of the above

13. Considering the graph, which of the following is true?



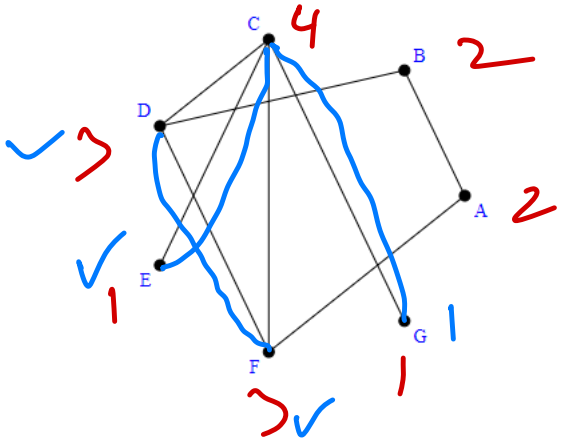
a. The graph has both an Euler circuit and a Hamiltonian circuit.

b. The graph has an Euler circuit, but not a Hamiltonian circuit

c. The graph has a Hamiltonian circuit, but not an Euler circuit.

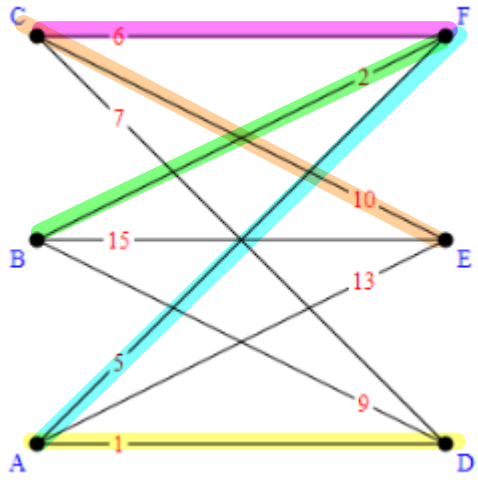
d. The graph has neither an Euler circuit nor a Hamiltonian circuit.

14. What is the least number of edges that must be duplicated to Eulerize the graph below.



3 min edges

15. Given the weighted graph below, apply Kruskal's algorithm to find a minimal spanning tree.



Graph has 6 vertice so tree needs 5 edges.

1) Choose cheapest edge: AD
2) BF
3) AF
4) CF
5) CE

1 + 2 + 5 + 6 + 10

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