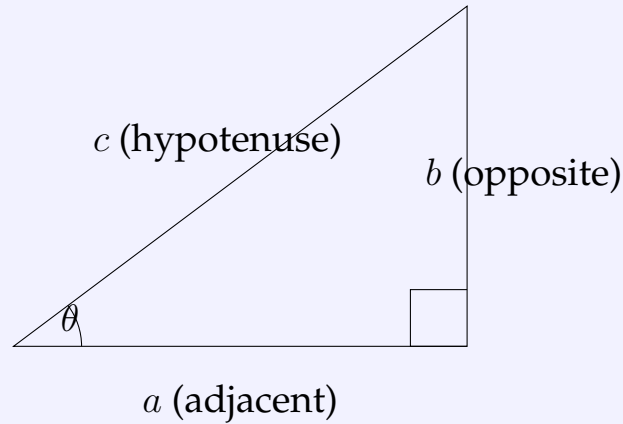


Right Triangle Trigonometry

Definition 4.3.1

In a right triangle with acute angle θ :
 latex



For right triangles, the Pythagorean Theorem applies: $c^2 = a^2 + b^2$, where c is the hypotenuse.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{b}{a}$$

$$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{c}{b}$$

$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{c}{a}$$

$$\cot \theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{a}{b}$$

Fundamental Identities

Reciprocal Identities

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\sin(\theta) = \frac{1}{\csc(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\cos(\theta) = \frac{1}{\sec(\theta)}$$

$$\cot(\theta) = \frac{1}{\tan(\theta)}$$

$$\tan(\theta) = \frac{1}{\cot(\theta)}$$

Quotient Identities

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

Pythagorean Identities

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

$$\cot^2(\theta) + 1 = \csc^2(\theta)$$

Even-Odd Properties

$$\sin(-\theta) = -\sin(\theta) \text{ (odd)}$$

$$\cos(-\theta) = \cos(\theta) \text{ (even)}$$

$$\tan(-\theta) = -\tan(\theta) \text{ (odd)}$$

$$\csc(-\theta) = -\csc(\theta)$$

$$\sec(-\theta) = \sec(\theta)$$

$$\cot(-\theta) = -\cot(\theta)$$

Pythagorean identities Remark:

Example 1. In the following examples find the exact values of the six trigonometric functions of the acute angle θ shown in the figure. Use the Pythagorean Theorem to determine the third side of the triangle.

Example Problem 1

Given: $c = 13, b = 5$.

Example Problem 2

Given: $a = 3, b = 8$.

Example 2. Sketch a right triangle corresponding to the trigonometric function of the acute angle θ . Use the Pythagorean Theorem to determine the third side of the triangle and then find the values of the other five trigonometric functions of θ .

Example Problem 1

Given: $\sec(\theta) = 2$

Example Problem 2

Given: $\tan(\theta) = 5$

Cofunction Identities

Cofunctions of complementary angles (sum to 90° or π_2 radians) are equal.

$$\sin(\theta) = \cos(90^\circ - \theta)$$

$$\cos(\theta) = \sin(90^\circ - \theta)$$

$$\tan(\theta) = \cot(90^\circ - \theta)$$

$$\cot(\theta) = \tan(90^\circ - \theta)$$

$$\sec(\theta) = \csc(90^\circ - \theta)$$

$$\csc(\theta) = \sec(90^\circ - \theta)$$

Example 3. Use the function value(s) and the trigonometric identities to evaluate each trigonometric function.

Example Problem 1

Given: $\sin(30^\circ) = \frac{1}{2}$, $\cos(30^\circ) = \frac{\sqrt{3}}{2}$.

(a) $\tan(30^\circ) =$

(b) $\sin(60^\circ) =$

(c) $\cos(60^\circ) =$

(d) $\cot(30^\circ) =$

Example Problem 2

Given: $\csc(\theta) = 5$, $\sec(\theta) = \frac{5\sqrt{6}}{12}$

(a) $\sin(\theta) =$

(b) $\cos(\theta) =$

(c) $\tan(\theta) =$

(d) $\sec(90^\circ - \theta) =$

Example Problem 3

Given: $\cot(\theta) = 3$

Example 4. Find the exact values of the indicated variables.

Example Problem 1

Find y and r in a right triangle with angle 30° , adjacent side = 16

