Math 142: Wednesday, Oct 29th, Day 20 Notes

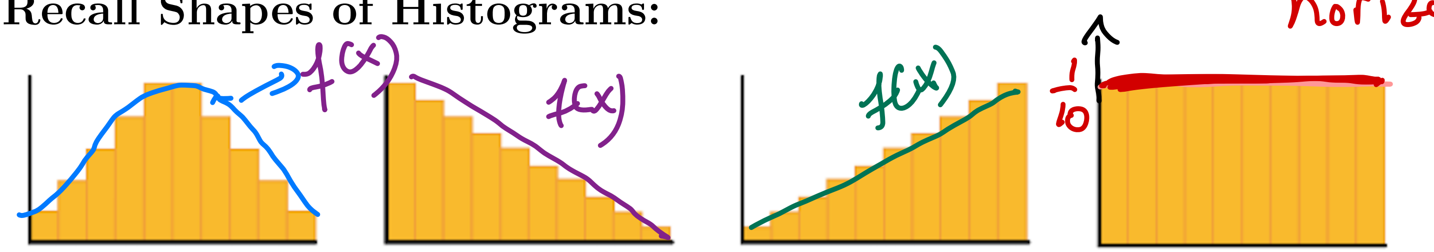
Agenda for Wednesday, Oct 29 2024

Coming up

- Student Hours Thursday 1-2:15pm in C162
- HW 13 due today
- Excel Assignment 3 due Today (Video posted in Canvas)
- Breakfast at Tutoring - Monday, November 4th,8 am - 11 am, Grayslake Tutoring Center (L135)

Recall Shapes of Histograms:

horizontal

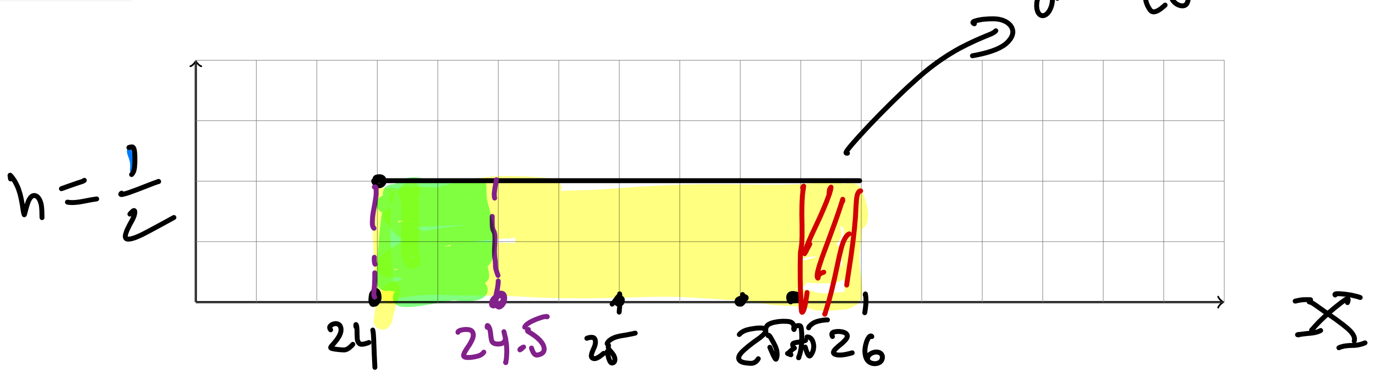


normal
(symmetric & unimodal)
bell curve
probability density function

uniform
$f(x) = \frac{1}{10}$

Uniform Distributions

Example 1. A manufacturer produces 25 pound dumbbell. The lowest actual weight is 24 pounds and the highest is 26 pounds. Each weight is equally likely so the distributions of weights is **uniform**.



1. Let X be the weight of the dumbell. Is X discrete or continuous random variable? What are the possible values?

X can be any between 24 lbs and 26 lbs.

2. What is the formula for probability density function $f(x)$?

$f(x) = \frac{1}{2}$

$A = (\text{base})\ \text{height}$

$1 = (26-24)\ h$

$1 = 2 \cdot h \Rightarrow h = \frac{1}{2}$

3. What is the probability that a weight is exactly 25.375 lbs?

0

4. What is the probability that the weight is less than 24.5 lbs?

$P(x < 24.5) =$ area of green rectangle $= (0.5)(1/2) = 0.25$

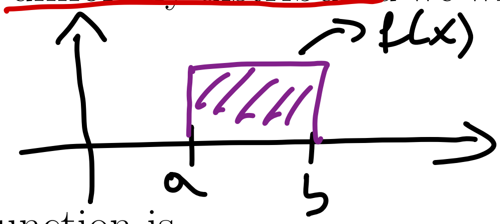
5. What is the probability that the weight is more than 25.75 lbs?

$P(x > 25.75) = (.25)(1/2) = 0.125$

Notation and Properties of the Uniform Distribution

- When a continuous random variable, X, is uniformly distributed we write

$X \sim U(\underbrace{a}_{\text{lower}}, \underbrace{b}_{\text{upper value}})$



- The formula for the probability density function is

$$f(x) = \frac{1}{b-a}$$

Area $= b \cdot h$

$1 = (b-a)\,h$

$h = \frac{1}{b-a}$

ex: $a=2$, $b=5$ $\quad f(x) = \frac{1}{5-2} = 1/3$

- The mean of a uniform probability distribution is

$$\mu = \frac{a+b}{2}$$

- The standard deviation

$$\sigma = \frac{b-a}{\sqrt{12}}$$

- Probability = Area or rectangle = (Base)(height)

Example 2. Calculate the mean and standard deviation for the previous example.

$$\mu = \frac{a+b}{2} = \frac{24+26}{2} = 25 \text{ lbs}$$

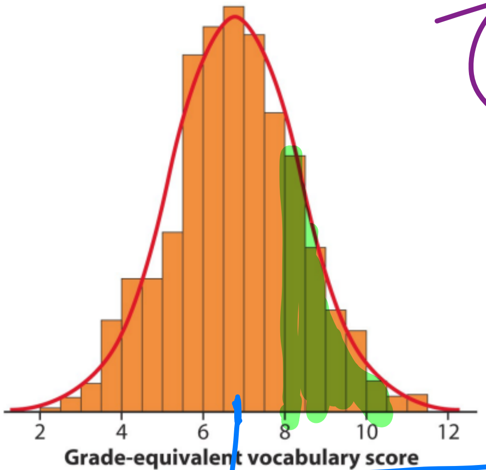
$$\sigma = \frac{b-a}{\sqrt{12}} = \frac{26-24}{\sqrt{12}} = \frac{2}{\sqrt{12}} = 0.577 \text{ lbs}$$

Normal Distributions

A **normal distribution** occurs naturally in many situations. For example, the normal distribution aka the bell curve, is seen in tests such as the SAT and GRE.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

~~no need to know~~

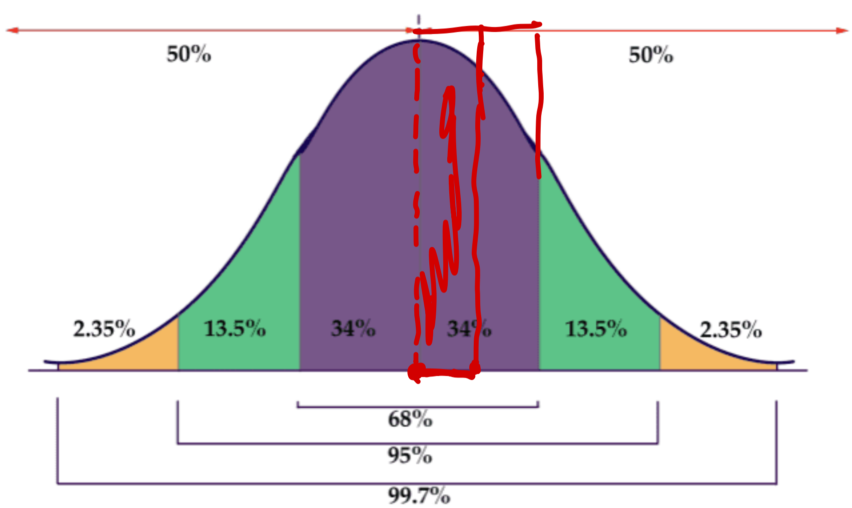


mean $= \mu =$ median $=$ mode

Many other types of data follow this bell-shaped pattern including salaries, measurement errors, heights and weights.

Notation and Properties of normal curve

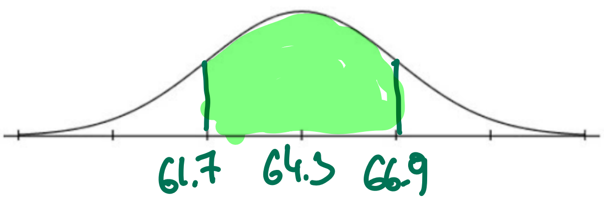
- Bell shaped, Symmetrical and Unimodal – it has one “peak”
- Mean and median are equal; both are located at the center of the distribution
- Total area under the curve is 1
- Empirical Rule holds



- Probability = area under the curve (we will use calculator to find it)

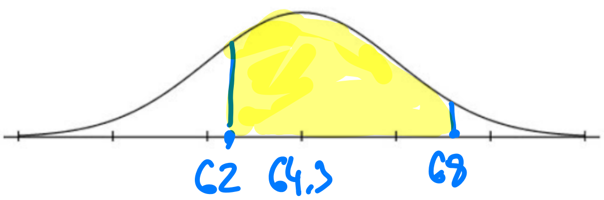
Example 3. A survey was conducted to measure the heights of U.S. women in the military. Respondents were grouped by age. In the 20 to 29 age group, the heights were **normally distributed**, with a **mean of 64.3 inches** and a **standard deviation of 2.6 inches**. A study participant is randomly selected. $X \sim N(\mu, \sigma)$, $\mu = 64.3$, $\sigma = 2.6$

a) Find the probability that her height is between 61.7 and 66.9 inches.



$P(61.7 < X < 66.9) = .68$

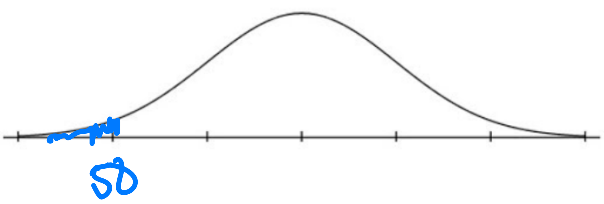
b) Find the probability that her height is between 62 and 68 inches.



$P(62 < X < 68) =$

$= \text{normalcdf}(62, 68, 64.3, 2.6)$

c) Find the probability that her height is less than 58 inches.

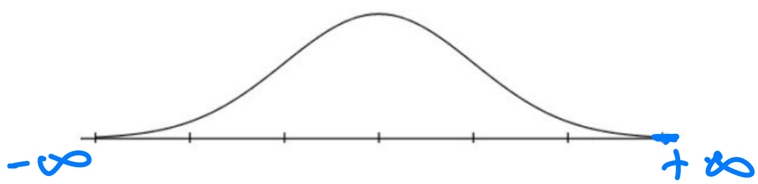


$P(X < 58) =$

$\text{normalcdf}(-1E99, 58, 64.3, 2.6)$

$= .008$

d) Find the probability that her height is more than 6ft.



$P(X > 72) =$

$\text{normalcdf}(72, 1E99, 64.3, 2.6)$

$=$

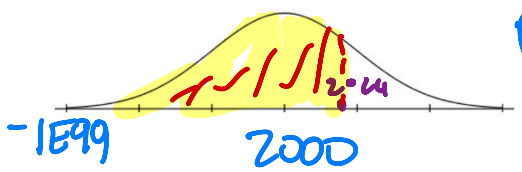
Calculator Formula to find normal probabilities:

- $P(a < X < b) = \text{normalcdf}(\text{lower val } a, \text{ upper value } b, \text{ mean } \mu, \text{ st. dev } \sigma)$
- $P(X < \text{upper}) = \text{normalcdf}(-1E99, \text{upper}, \mu, \sigma)$
- $P(X > \text{lower}) = \text{normalcdf}(\text{lower}, 1E99, \mu, \sigma)$

Example 4. The life spans of batteries are **normally distributed**, with a **mean of 2000 hours** and a **standard deviation of 30 hours**. Write the probability in correct notation. Draw the normal curve and shade the correct area. Find each probability using calculator.

a) What is the probability that the lifespan of these batteries is less than 2024 hours?

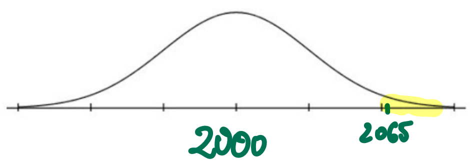
$X \sim N(2000, 30)$



$P(X<2024) = \text{normalcdf}(-1E99, 2024, 2000, 30)$

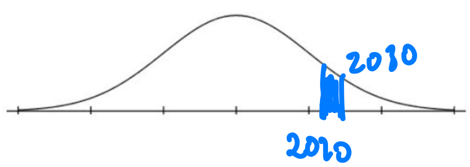
$\boxed{0.79}$

b) What percent of batteries have a life span that is more than 2065 hours?



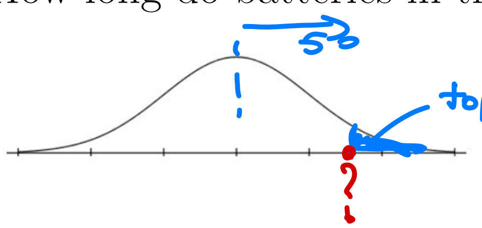
$P(X>2065) = \text{normalcdf}(2065, 1E99, 2000, 30)$
$= 0.015$

c) What is the probability that the lifespan of these batteries is is between 2020 and 2030 hours?



$P(2020<X<2030) = \text{normalcdf}(2020, 2030, 2000, 30)$
$= 0.094$

d) How long do batteries in the top 10% last?



given area
find the x value

Calculator Formula to find inverse normal probabilities:

invNorm(area to the left , mean , st dev)

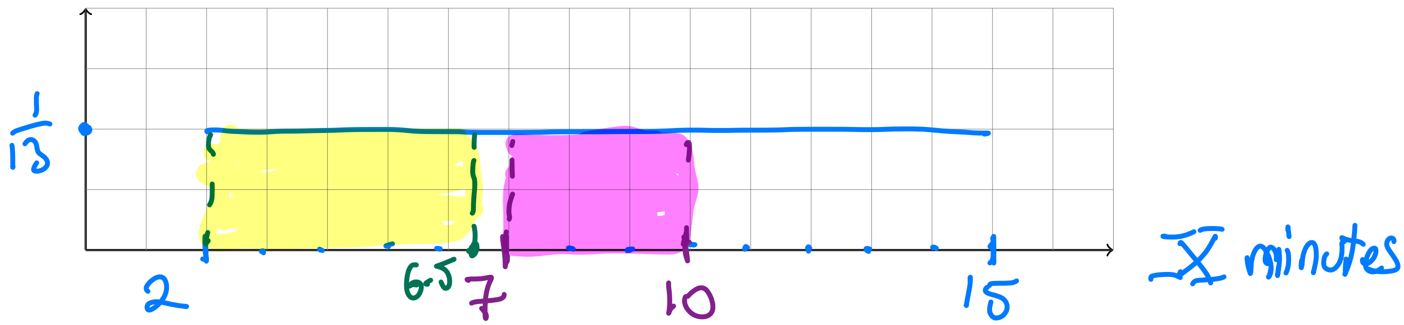
invNorm(.9 , 2000 , 30) = 2038 hours

Ans: The top 10% batteries last more than 2037 hours

Check your answers

Practice in Groups (or on your own at home if we have no time left in class)

Example 5. Suppose the distribution of arrival times is **uniform** with the minimum of 2 minutes and maximum of 15 minutes for Fire trucks reaching the scene after being called.



a) What is the formula for probability density function $f(x)$?

$$f(x) = \frac{1}{b-a} = \frac{1}{15-2} = \frac{1}{13}$$

b) What is the probability that a fire truck will reach the scene within 7 to 10 minutes?

$$P(7 < X < 10) = \text{Area} = b \cdot h = (3)\frac{1}{13} = \frac{3}{13}$$

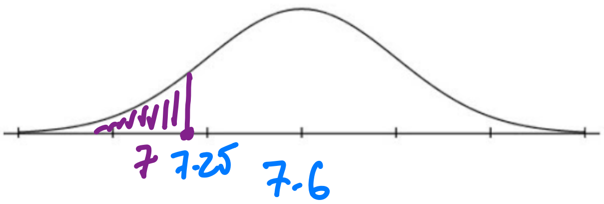
c) What is the probability that a fire truck will reach fire less than 6.5 minutes?

$$P(X < 6.5) = \text{Area} = b \cdot h = (6.5-2)\frac{1}{13} = \frac{4.5}{13} = 0.346$$

Example 6. The amounts of time employees at a large corporation work each day are normally distributed, with a mean of 7.6 hours and a standard deviation of .35 hour. Find the following probabilities. $X \sim N(7.6, .35)$

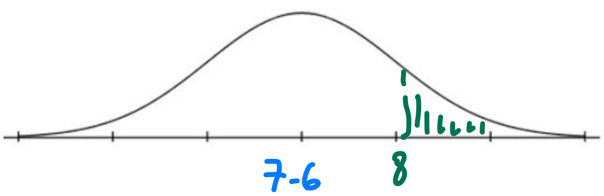
a) What is the probability that a person works less than 7 hours?

$P(x<7) = \text{normalcdf}(-1E99, 7, 7.6, .35)$
$= 0.043$



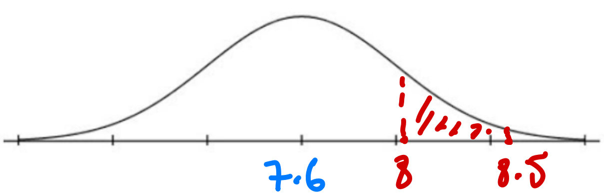
b) What is the probability that a person works more than 8 hours?

$P(x>8) = \text{normalcdf}(8, 1E99, 7.6, .35)$
$= 0.127$

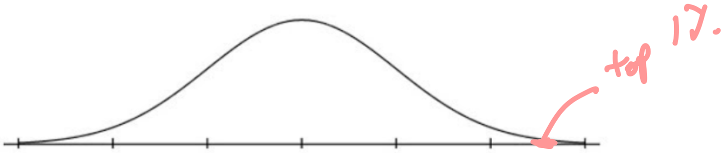


c) What is the probability that a person works between 8 and 8.5 hours?

$P(8<x<8.5) = \text{normalcdf}(8, 8.5, 7.6, .35$
$= 0.121$



d) The top 1% person are considered workaholics. What is the least amount of hours you would work to be considered an workaholic?



$\text{invNorm}(.99, 7.6, .35) = 8.414$ hours

Someone who works more than 8.414 is considered an workaholic.