Math 140: Tuesday Aug 27, Day 3 Notes

Agenda for Tuesday, Aug 27,2024

Reminders

- ! Turn in Written HW 1 by Thursday (upload in Canvas or bring to class)
- Office Hour **today 1-2:15pm** in C162
- **First Group Quiz** on Thursday, August 29th in class (you can use your handwritten notes)
- **Student Resource Fair** Thursday, August 29 from 11:30 a.m.-1:30 p.m. & 4:30-6 p.m. Student Commons/Student Street

Section 1.2 Euler Path and Circuits

Recall: **Euler's Circuit Theorem:**

- connected & each vertex has even degree, then we have at least one Euler circuit.

Euler's Path Theorem:

- connected & if it has exactly two odd degree vertices, then it has an Euler path. The path should start at one of the odd vertex & end at other one.

Example 1. For each of the three graphs below choose from one of the following answers and provide a short explanation for your answer.

A) The graph has an Euler circuit.

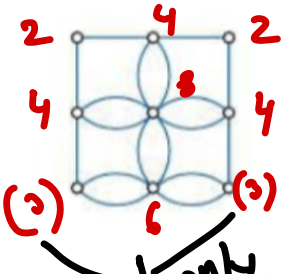
B) The graph has an Euler path.

C) The graph has neither an Euler circuit nor an Euler path.

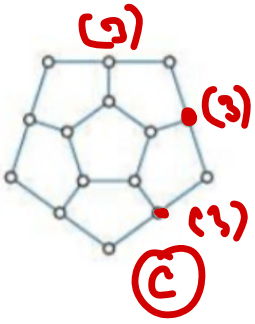
D) The graph may or may not have an Euler circuit.

E) The graph may or may not have an Euler path.

(a)



(b)



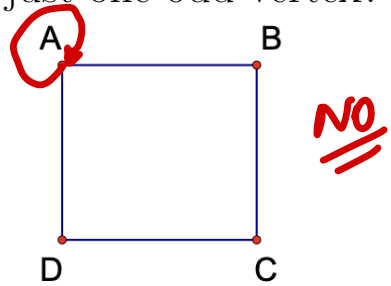
(c) A graph with eight vertices: six vertices of degree 2 and two vertices of degree 3.

(B) only two odd degree vertex

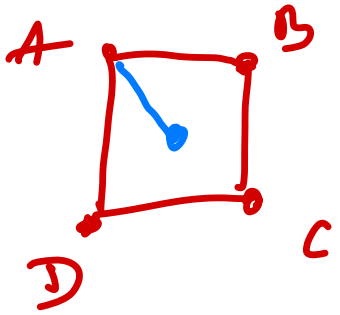
(C)

(E) doesn't say connected graph.

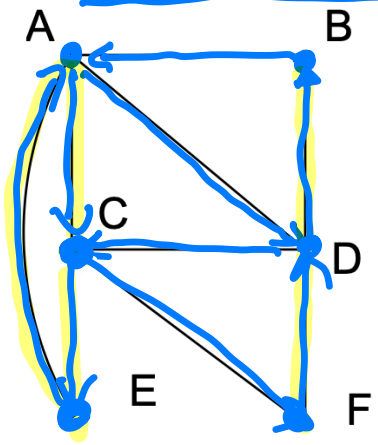
Example 2. Is it possible to add an edge so that you get just one odd vertex?



NO



Example 3. Find an Euler Circuit in the following graph using Fleury's Algorithm.

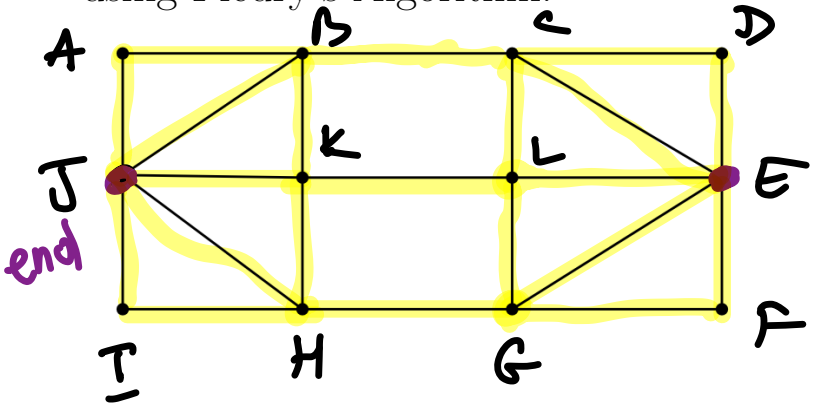


1) ✓ yes has Euler circuit

2) start at B

3) BADCEACFDB

Example 4. Find an Euler path in the following graph using Fleury's Algorithm.



1) yes J and E

2) start at E

3) (start) E

end J

EFGEDCELCBAJBKJIHGL

KHJ

Euler's Sum of Degrees Theorem

- The sum of the degrees of all the vertices of a graph equals twice the number edge. (check it)
- A graph has always an even number of odd vertices (0, 2, 4, 6, .. odd vertices)

Fleury's algorithm for finding an Euler Circuit or Path

1. Determine the number of odd vertices in the graph. (If the graph is not connected, there is no Euler Circuit or Euler Path.)

zero ← 0 odd vertices we have an Euler circuit | 2 odd vertices we have an Euler path | otherwise no Euler circuit or path

2. Select a starting vertex.

Euler circuit start at any vertex | Euler path start at one of the odd vertex and end at the other.

3. At each step, if you have a choice, don't choose a bridge of the yet-to-be-traveled part of the graph.

4. End.

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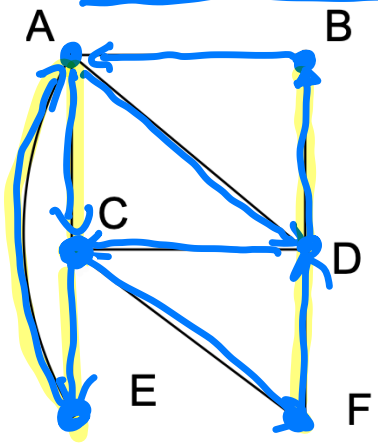

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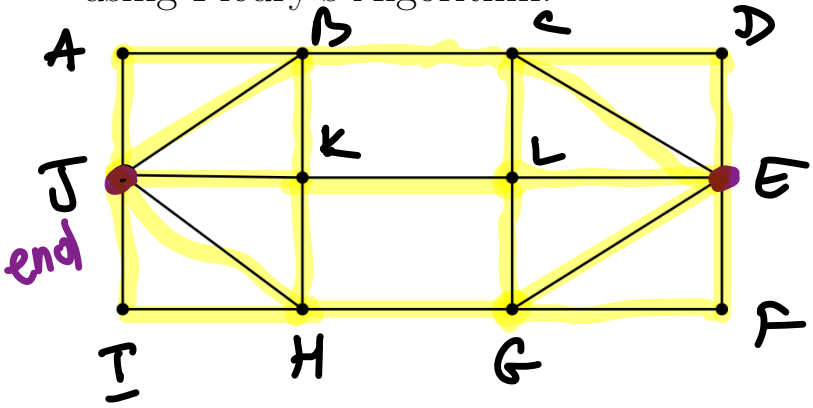


1) yes J and E

2) start at E

3) (start) E

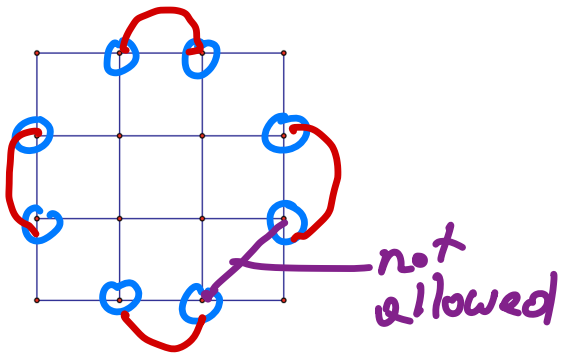
end J

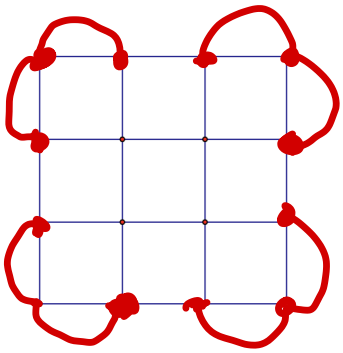
EFGEDCELCBAJBKJIHGL

KHJ

What do you do if a graph doesn't have an Euler Circuit or an Euler Path but you still need to travel all the edges?

Eulerization means creating a new graph that has an Euler circuit by adding edges to existing edges with the purpose of having no odd vertices.





not optimal!

Semi-Eulerization means creating a graph that has an Euler path by adding edges to existing edges such that there are exactly 2 odd degree vertices.