Day 13 Notes
2.6 Transformations of Graphs -continued

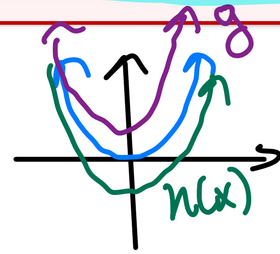
1. Transformations of functions

In this section, we'll discuss some ways to graph more complicated popular functions. For example, we will find a quick way to graph the function $f(x) = -(x+2)^2 + 5$ just by knowing that this graph is the graph of x^2 (which we call the **parent** or **basic** function) transformed in some way. These transformation can be rigid or non-rigid.
A **rigid transformation** changes the location of the function in a coordinate plane, but leaves the size and shape of the graph unchanged.
A **non-rigid transformation** changes the size and/or shape of the graph.

Recall: $f(x) = x^2$

$g(x) = x^2 + 2$

$h(x) = x^2 - 1$

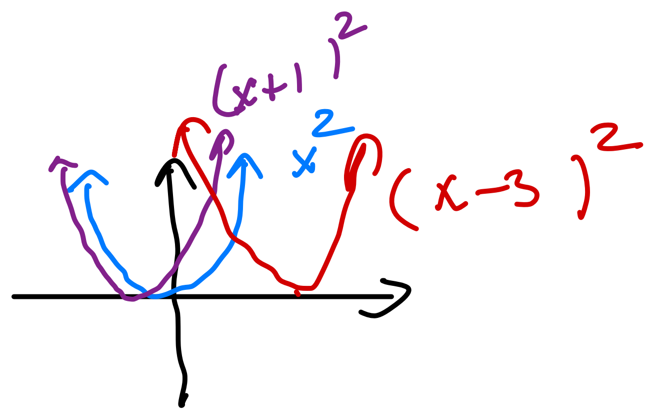


up/down

Left or right

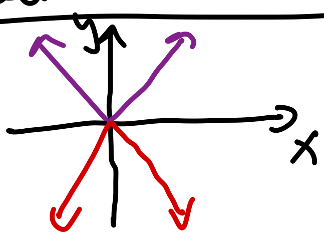
$f(x) = x^2$

$g(x) = (x+1)^2$ ↳ left one unit



$h(x) = (x-3)^2$ right 3 units

Reflection over x axis

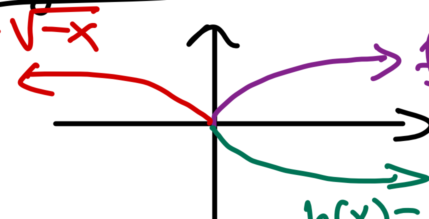


$f(x) = |x|$

$f(x) = -|x|$

Reflection over y axis

$g(x) = \sqrt{-x}$



$f(x) = \sqrt{x}$

$h(x) = -\sqrt{x}$

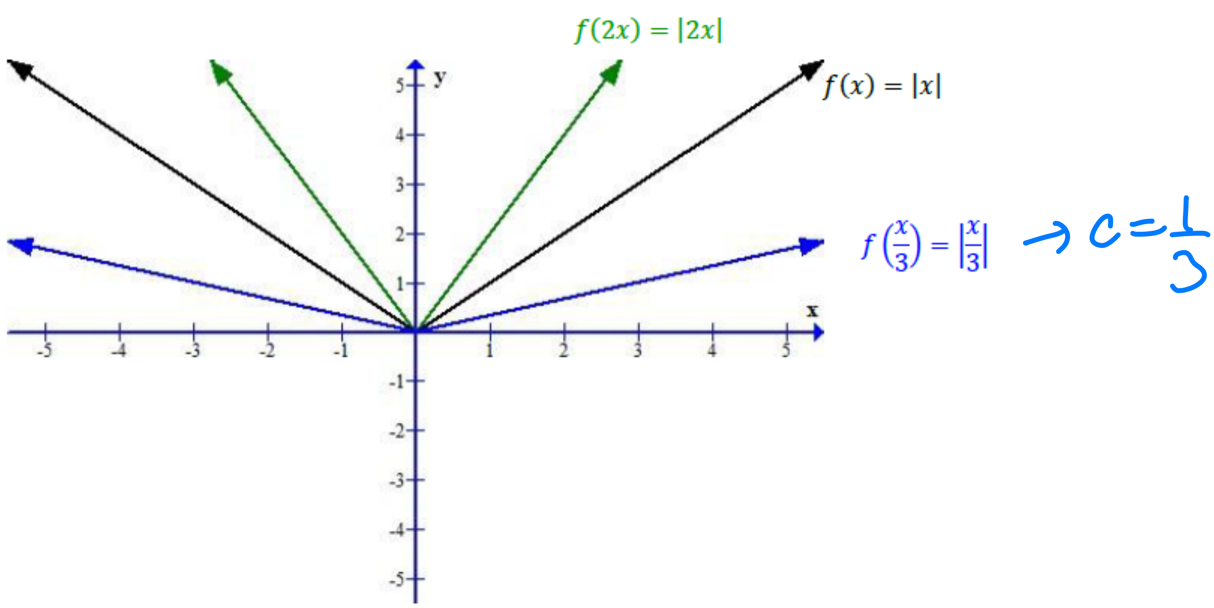
1 Stretching (non-rigid transformation)

Horizontal stretch/shrink

For any function $f(x)$, if c is multiplied to the variable of the function then the graph of the function will undergo a horizontal stretching or compression. $f(x)=|x|$ $|2x|$

- When the function becomes $y = f(cx)$ and $0 < c < 1$, a horizontal stretching of the graph of will occur. $|\frac{1}{2}x|$
- Graphically, a horizontal stretching pulls the graph of $y = f(x)$ away from the y-axis.
- When $c > 1$ in the function $y = f(cx)$, a horizontal shrinking of the graph of $y = f(x)$ will occur.
- A horizontal shrinking pushes the graph of toward the y-axis.
- In general, a horizontal stretching or shrinking means that every point (x, y) on the graph of is transformed to $\left(\frac{x}{c}, y\right)$ on the graph of $y = f(cx)$.

Example 1.
Horizontal Stretch and shrink



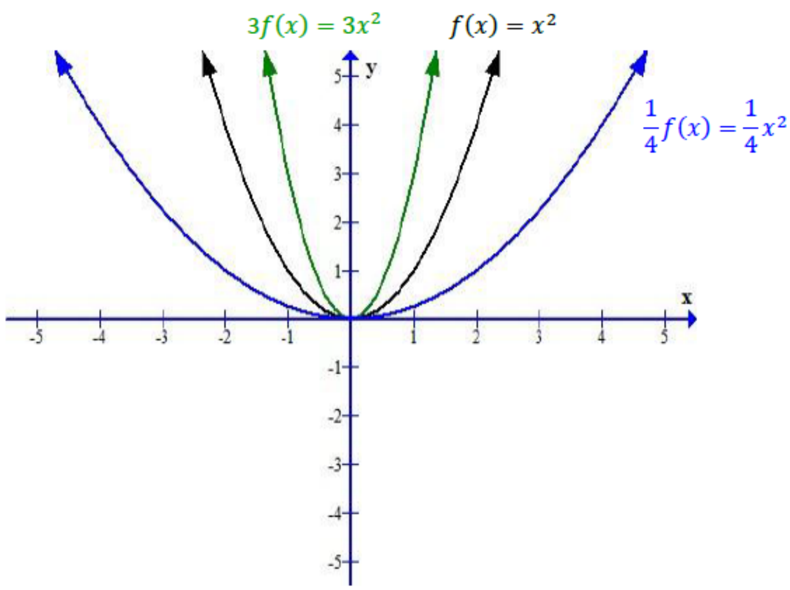
Vertical stretching and shrinking: non-rigid

$f(x) = x^2$ $g(x) = 2x^2$
$h(x) = \frac{1}{3}x^2$

- If c is multiplied to the function then the graph of the function will undergo a vertical stretching or compression.
- So when the function becomes $y = cf(x)$ and $0 < c < 1$, a vertical shrinking of the graph of will occur.

compress $\quad$ $1/2$, $3/4$, $1/3$

- Graphically, a vertical shrinking pulls the graph of $y = f(x)$ toward the x-axis.
- When $c > 1$ in the function $y = cf(x)$, a vertical stretching of the graph of $y = f(x)$ will occur.
- A vertical stretching pushes the graph of $y = f(x)$ away from the x-axis.
- In general, a vertical stretching or shrinking means that every point (x, y) on the graph of $f(x)$ is transformed to (x, cy) on the graph of $y = cf(x)$.



2 Using transformations to graph functions

Transformations can be combined within the same function so that one graph can be shifted, stretched, and reflected. If a function contains more than one transformation, **perform the transformations in the following order** to graph the function:

1. Horizontal translation
2. Stretching or shrinking
3. Reflecting
4. Vertical translation

Summary

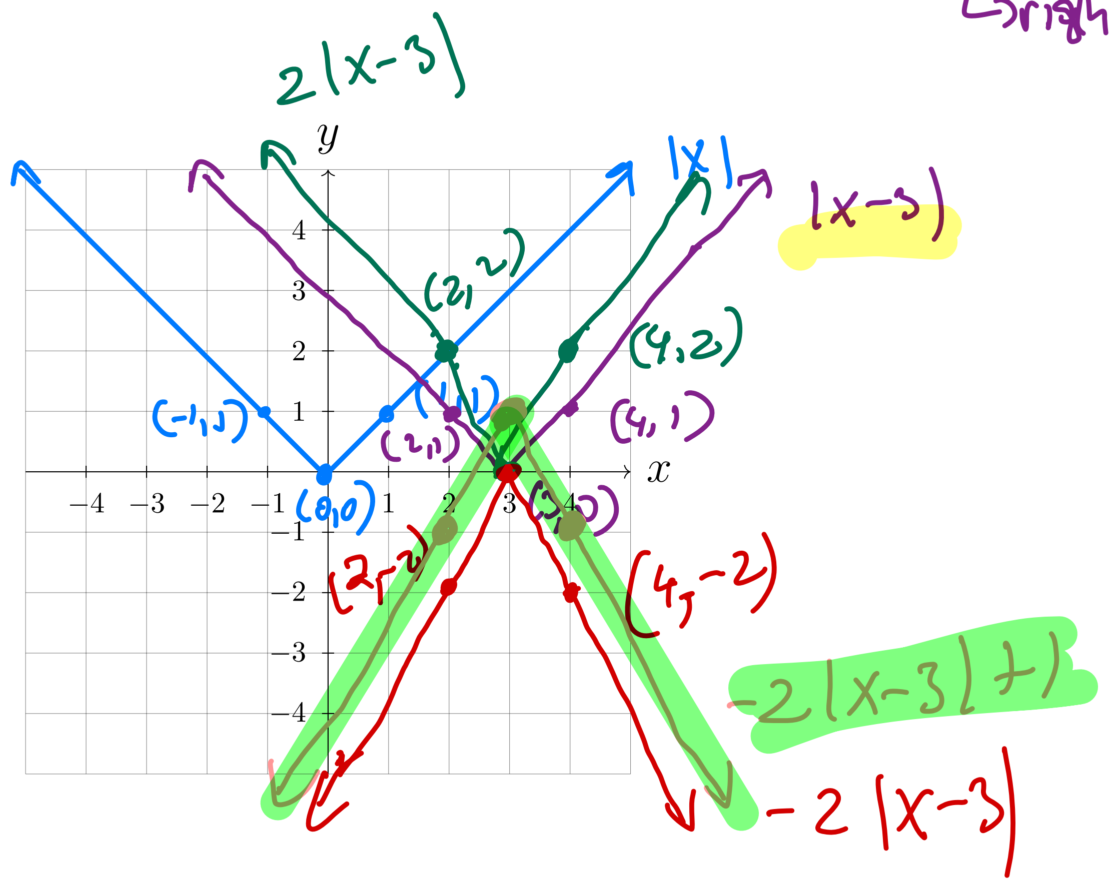
Transformations of the graphs of functions	
$f(x)+c$	shift $f(x)$ up c units
$f(x)-c$	shift $f(x)$ down c units
$f(x+c)$	shift $f(x)$ left c units
$f(x-c)$	shift $f(x)$ right c units
$f(-x)$	reflect $f(x)$ about the y-axis
$-f(x)$	reflect $f(x)$ about the x-axis
$cf(x)$	When $0 < c < 1$ – vertical shrinking of $f(x)$ When $c > 1$ – vertical stretching of $f(x)$ **Multiply the y values by c**
$f(cx)$	When $0 < c < 1$ – horizontal stretching of $f(x)$ When $c > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by c**

Practice: Graph the parent function $|x|$ and the function $f(x) = -2|x-3|+1$

↳ rigth 3

Follow three points.

Order of trans
1) Left or Right ✓
2) Stretch/Shrink ✓
3) Reflection x axis
4) Up/Down up!



2) vertical stretch with $c=2$

$(3,0) \longrightarrow (3, 0\cdot 2) = (3,0)$

$(4,1) \longrightarrow (4, 2\cdot 1) = (4,2)$

$(2,1) \rightarrow (2, 2\cdot 1) = (2,2)$

3) reflection xaxis

$(3,0) \rightarrow (3,0)$

$(4,2) \rightarrow (4,-2)$

$(2,2) \rightarrow (2,-2)$

4) up 1 unit

$(3,0) \rightarrow (3,1)$

$(4,-2) \rightarrow (4,-1)$

$(2,-2) \rightarrow (2,-1)$

3 Section 2.7 Analyzing graphs of functions and Piecewise defined functions

$f(x) = x^2+2$
$f(x) = 2|x-1|$

2. Piecewise defined functions

Graph the function

$$f(x) = \begin{cases} x^2+2 & x \le -1 \\ x & -1 < x \le 1 \\ -1 & 1 < x \end{cases}$$

(parabola)
(identity)
(constant)

First evaluate the following:
$f(0), f(-1), f(-2), f(3), \cancel{f(-1)}$

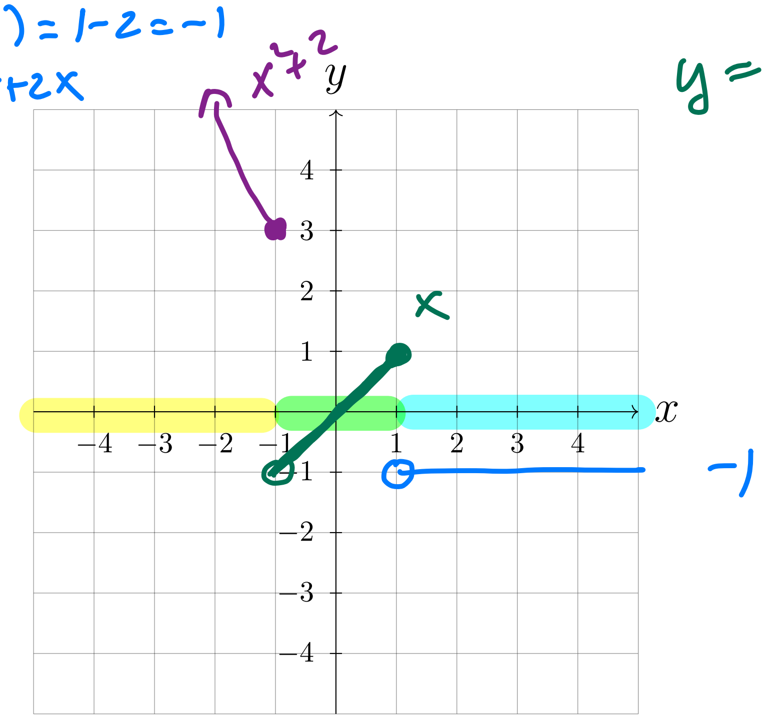
$f(-2) = (-2)^2 + (2)(-2) = 0$
piece 1: $f(x) = x^2+2x$

$f(0) = 0$
piece 2: $f(x) = x$

$f(3) = -1$
piece 3: $f(x) = -1$

$f(-1) = (-1)^2 + (2)(-1) = 1-2 = -1$
piece 1: $f(x) = x^2+2x$

$y = x$



Steps to graph a piecewise function:
1. Graph each piece on its given domain
2. Use solid dots (•) for included endpoints
3. Use open dots (○) for excluded endpoints $1<x$ (1 is excluded)
4. Check that the pieces connect as specified

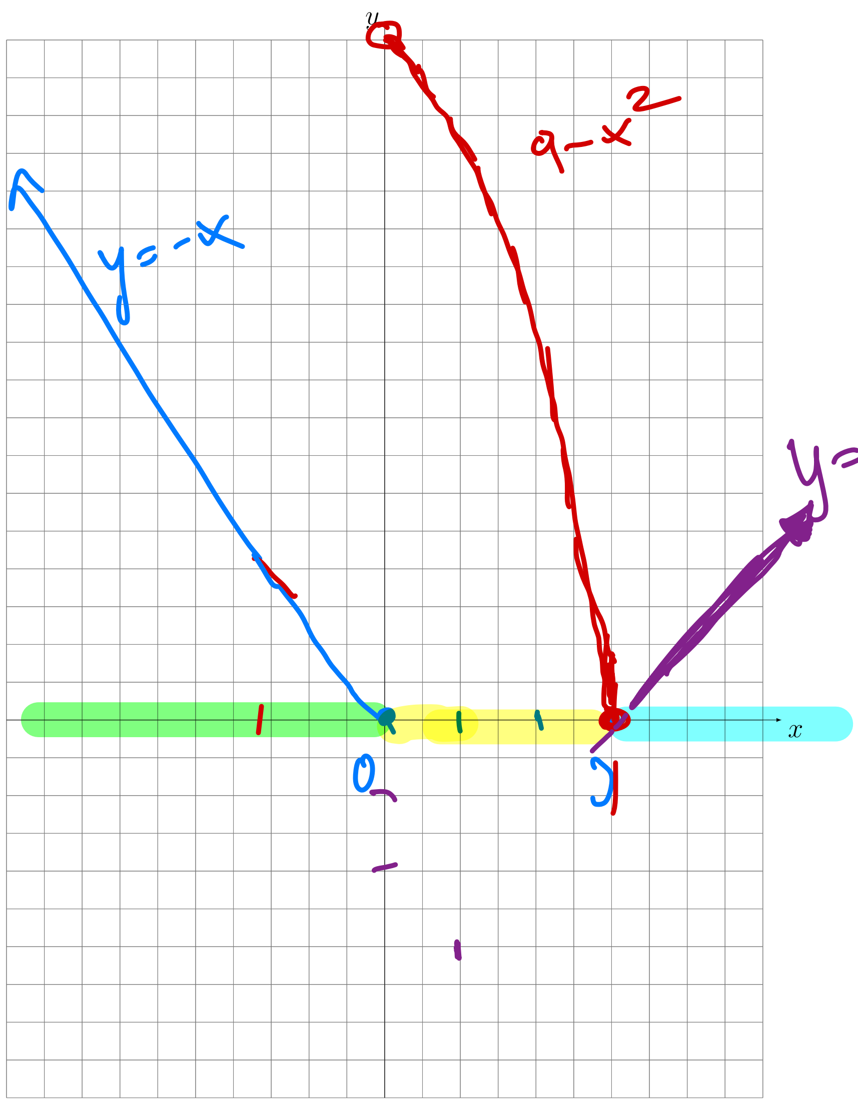
Practice: Graph the following function by plotting points.

$$f(x) = \begin{cases} -x & x \leq 0 \\ 9 - x^2 & 0 < x \leq 3 \\ x - 3 & 3 < x \end{cases}$$

line

parabola

line

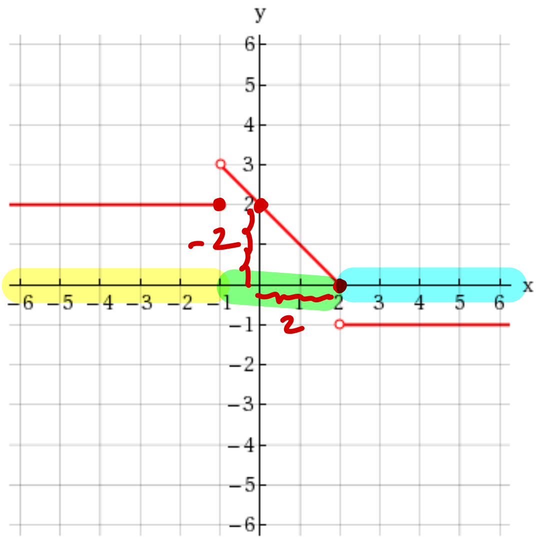


piece 1: $y=-x$
line w/ slope -1
and y int 0.

piece 2: $y=9-x^2$
x^2 reflect over x axis
up 9 units

piece 3: $y=x-3$
slope $m=1$
y int $=-3$

Practice: Find a formula for the function and state its domain and range.



$$f(x)=\begin{cases} 2, & x \le -1 \\ -x+2, & -1 < x \le 2 \\ -1, & x > 2 \end{cases}$$

$y = mx+b$

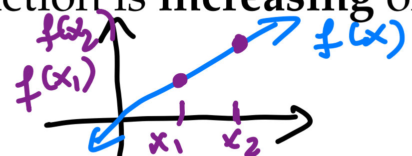
$y = mx+2$ $\quad m = \frac{\text{rise}}{\text{run}}$

$y = -x+2$ $\quad m = \frac{-2\text{ (down 2)}}{2\text{ (right 2)}}$

Increasing/decreasing/constant function

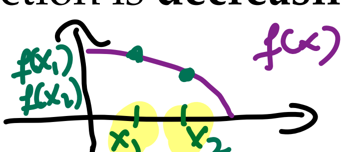
Let f be a function.

- A function is **increasing** on an interval if as x increases, $f(x)$ increases.



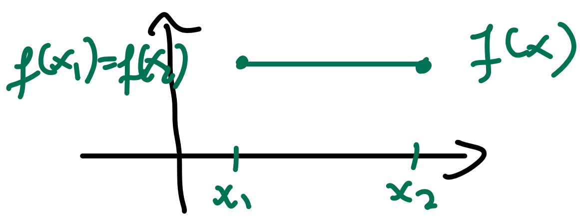
from left to right graph rises.

- A function is **decreasing** on an interval if as x increases, $f(x)$ decreases.



from left to right graph falls.

- A function is **constant** on an interval if $f(x)$ remains the same as x changes.



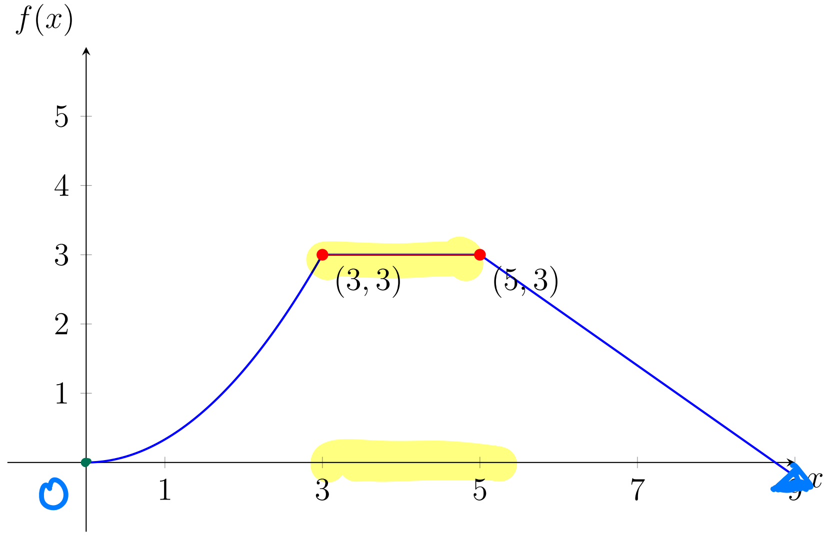


Figure 1: Graph of a function with increasing, constant, and decreasing regions

x interval

Example 2. Where is the function increasing/decreasing/constant?

increasing : $(0,3)$

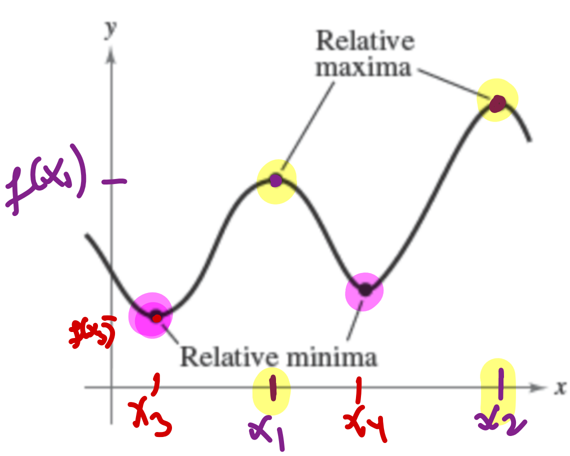
decreasing : $(5,\infty)$

constant: $(3,5)$

Relative (local) maxima/minima

For a function f:

- A point is a **local maximum** if $f(x)$ is greater at that point than at nearby points.
- A point is a **local minimum** if $f(x)$ is less at that point than at nearby points



x_2 – local max

x_3 and x_4 are local minima

At x_1 we will have a local max and its value is $f(x_1)$.

Even/odd functions

- A function is **even** if its graph is symmetric about the y-axis

$$f(-x) = f(x) \text{ for all } x \text{ in the domain}$$

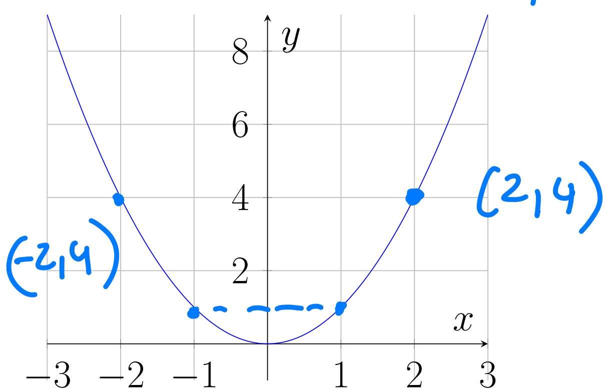
- A function is **odd** if its graph is symmetric about the origin

$f(-1) = -f(1)$
$-1 = -1$

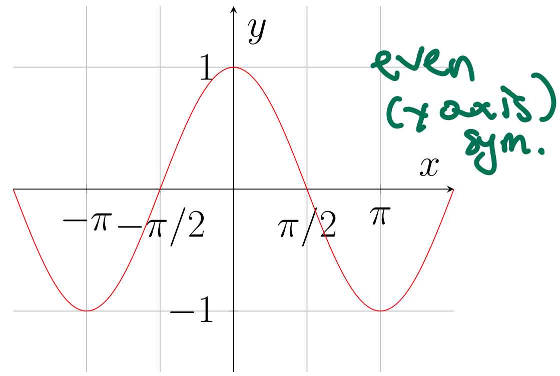
$$f(-x) = -f(x) \text{ for all } x \text{ in the domain}$$

Function: $f_1(x) = x^2$

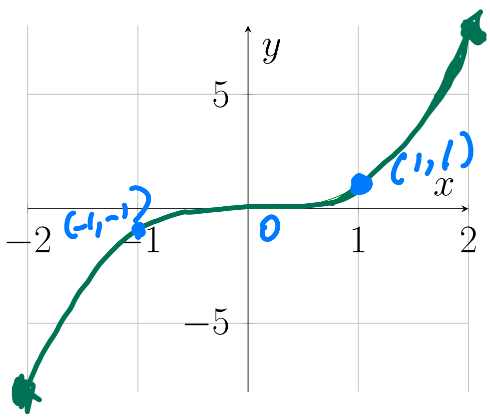
$f(-2) = (-2)^2$
$= 4$



Function: [redacted]

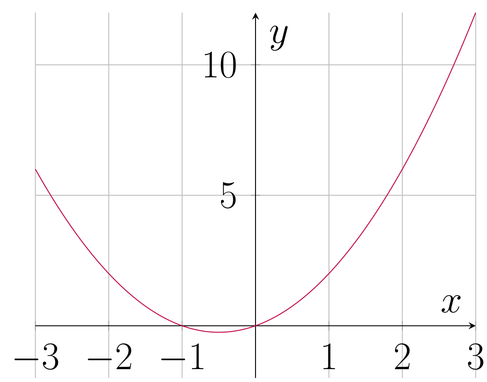


Function: $f_3(x) = x^3$



$x^{1/2} = \sqrt[2]{x^1}$

$f_4(x) = x^2 + x^1$



Example 3. Determine whether the function is even, odd or neither.

1. $f(x) = x^3 + 4x$
2. $g(x) = 3x - x^2$
3. $f(t) = 5t^{2/3}$

③ $f(-t) = 5(-t)^{2/3} = 5\sqrt[3]{(-t)^2} = 5\sqrt[3]{t^2}$
$= 5t^{2/3} = f(t)$ even

To see if a function is odd/even/neither calculate $f(-x)$.

① $\boxed{f(-x)} = (-x)^3 + 4(-x) = -x^3 - 4x = -(x^3+4x) = \boxed{-f(x)}$ odd

② $g(-x) = 3(-x) - (-x)^2 = -3x - x^2$ neither