Wednesday, April 22nd

WHW5 - submit if you haven't done so yet

WHW 6 - will be posted today

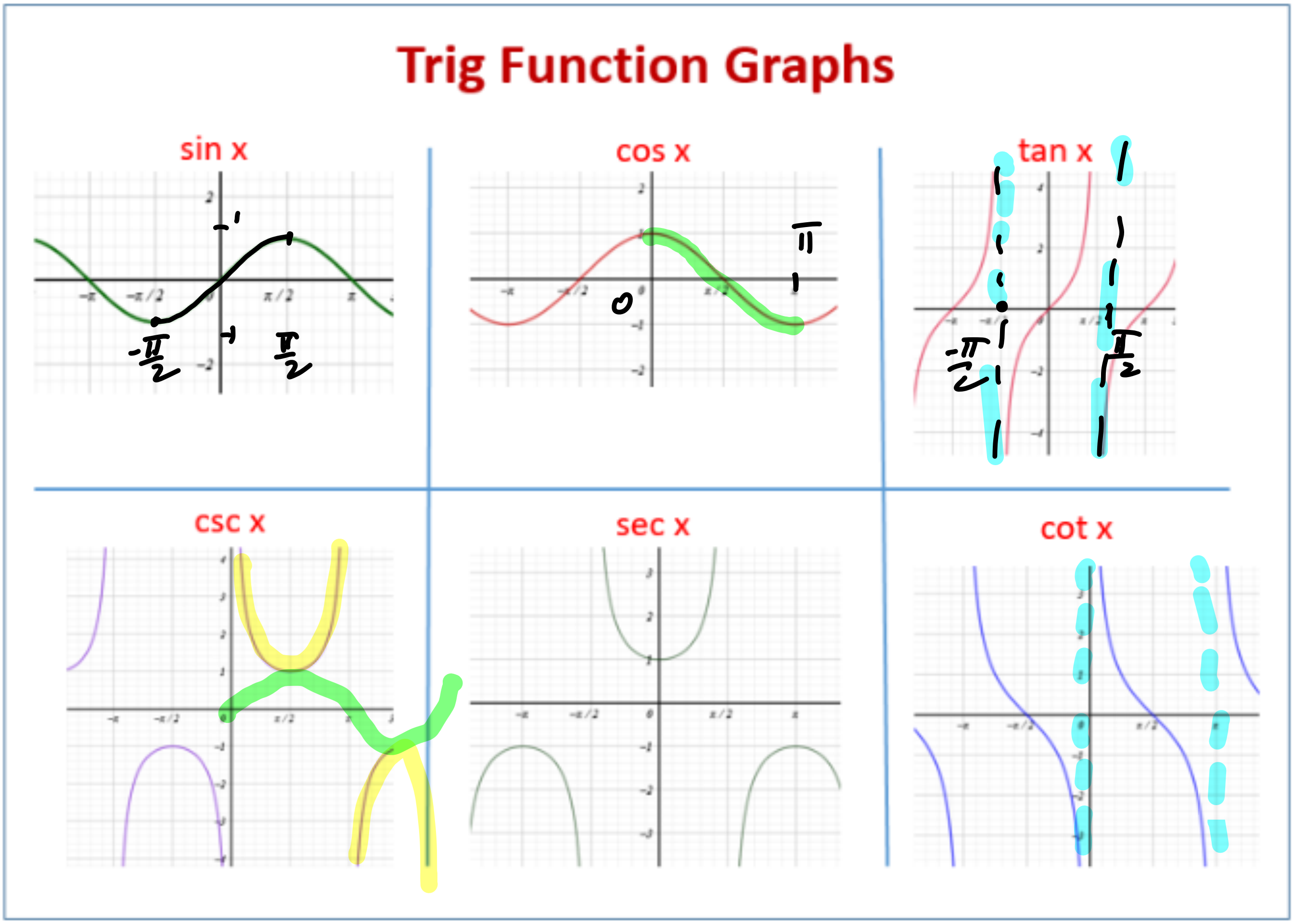
Exam 4 - next Wednesday April 30th

Final Exam Wed May 14th 12-1:50pm

Student Portal: Student Center
Manage classes
Final schedule (on left)

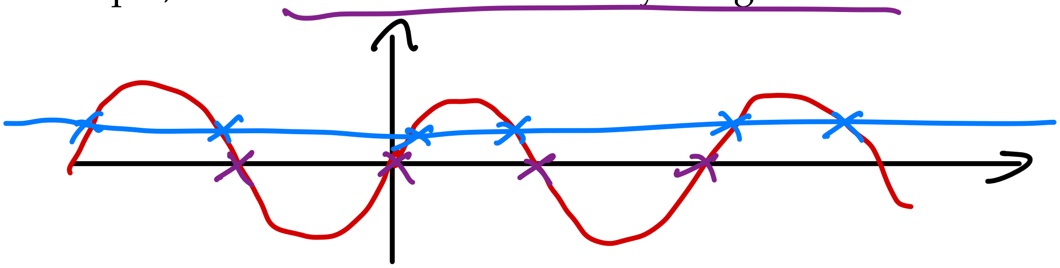
Recall Graphs of Sine, Cosine and Tangent Functions

The graphs of the basic trigonometric functions $\sin(x)$, $\cos(x)$, and $\tan(x)$ all fail the horizontal line test at infinitely many points.



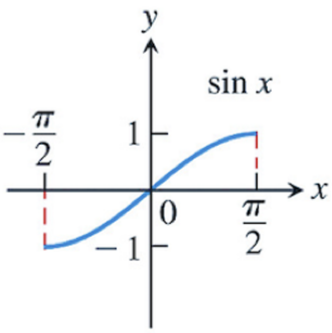
For example, $\sin x = 0$ for $x = 2n\pi$ for any integer n.

not one-to-one

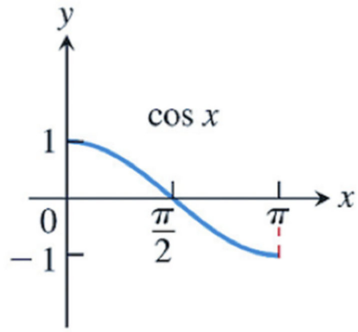


However, we can deal with this problem by **restricting the domain** of each function to an interval on which the function is $1-1$.

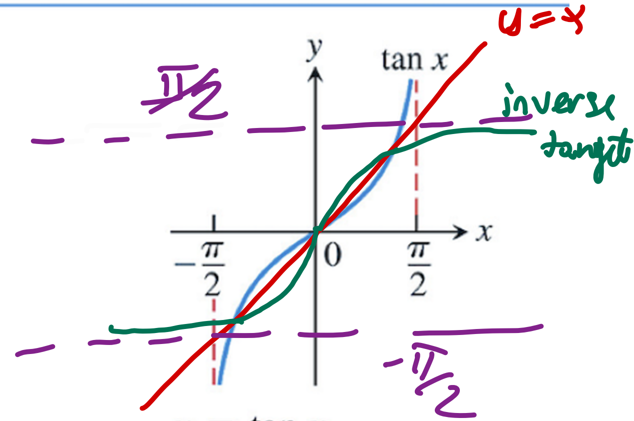
Domain restrictions that make the trigonometric functions one-to-one



$y = \sin x$
Domain: $[-\pi/2, \pi/2]$
Range: $[-1, 1]$



$y = \cos x$
Domain: $[0, \pi]$
Range: $[-1, 1]$



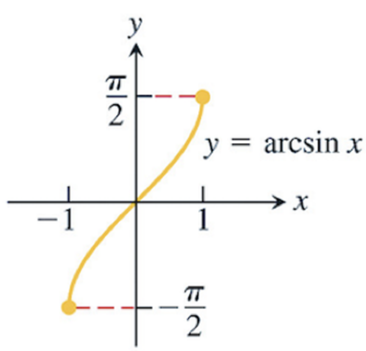
$y = \tan x$
Domain: $(-\pi/2, \pi/2)$
Range: $(-\infty, \infty)$

$f = \sin x$
$f^{-1} = \arcsin x = \sin^{-1} x$

Graphs of inverse trigonometric functions

Now that we are dealing with invertible functions, we can define inverses for each of these trig functions. We can denote the inverse of $\sin x$ on its restricted domain by $\sin^{-1}(x)$ or by $\arcsin(x)$.

Domain: $-1 \leq x \leq 1$
Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$



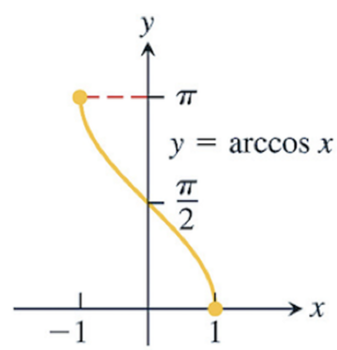
(a)

$y = \arcsin(x)$ input of inv
output of inverse

$\boxed{\sin y = x}$

$\log_2 x = 3$
$2^3 = x$

Domain: $-1 \leq x \leq 1$
Range: $0 \leq y \leq \pi$

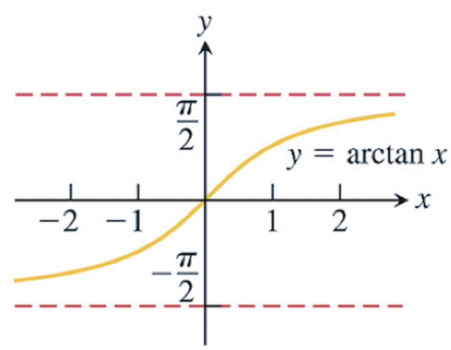


(b)

$y = \arccos(x)$

$\boxed{\cos y = x}$

Domain: $-\infty < x < \infty$
Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$



(c)

$y = \arctan(x)$

$\boxed{\tan y = x}$

Example 1. *Calculate the following, paying close attention to the domain and range of our inverse trigonometric functions:*

D: $[-1,1]$ R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

1. $\arcsin\left(-\frac{1}{2}\right) = ? = -\frac{\pi}{6}$

$\sin ? = -\frac{1}{2}$

$\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

D: $[-1,1]$ R: $[-\pi/2, \pi/2]$

2. $\arcsin(\sqrt{3}/2) = ?$

$\sin ? = \frac{\sqrt{3}}{2}$

↳ is in

$? = \pi/3$

$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

D: $[-1,1]$ R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

3. $\sin^{-1}(2) =$ undefined

D: $[-1,1]$ R: $[0,\pi]$

4. $\arccos\left(\frac{\sqrt{2}}{2}\right) = ? = \frac{\pi}{4}$

$\cos ? = \frac{\sqrt{2}}{2}$

$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

D: $[-1,1]$ R: $[0, \pi]$

5. $\cos^{-1}(-1) = ? = \pi$

$\cos ? = -1$

$\cos \pi = -1$

D: $(-\infty, \infty)$ R: $(-\pi/2, \pi/2)$

6. $\arctan 0 = ? = 0$

$\tan ? = 0$

$\tan 0 = 0$

D: $(-\infty, \infty)$ R: $(-\pi/2, \frac{\pi}{2})$

7. $\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

Compositions of Functions

$\sin(\arcsin(x))$ or $\cos(\arctan x)$
$\arcsin(\sin(x))$

When we compose trig functions and their inverses (i.e. sine composed with arcsine) we can make use of the inverse properties.

$f(f^{-1}(x)) = x$
$f^{-1}(f(x)) = x$

Inverse Properties

If $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$, then

$$\sin(\arcsin x) = x \quad \text{and} \quad \arcsin(\sin y) = y.$$

If $-1 \le x \le 1$ and $0 \le y \le \pi$, then

$$\cos(\arccos x) = x \quad \text{and} \quad \arccos(\cos y) = y.$$

If x is a real number and $-\pi/2 < y < \pi/2$, then

D: $(-\infty, \infty)$

$$\tan(\arctan x) = x \quad \text{and} \quad \arctan(\tan y) = y.$$

Caution!!! Keep in mind that these properties do not apply for arbitrary values of x and y. For instance,

$$\cancel{\arcsin}\left(\cancel{\sin}\frac{3\pi}{2}\right) \neq \frac{3\pi}{2}.$$

$\frac{3\pi}{2}$ is outside of $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$!

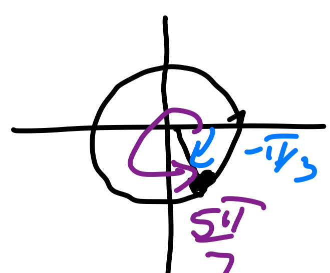
$\arcsin\left(\sin\frac{3\pi}{2}\right) = \arcsin(-1) = \,? = -\pi/2$

$\sin\,? = -1$

Example 2. *If possible, find the exact value.*

a. $\tan[\arctan(-5)] = -5$

b. $\arcsin\left(\sin\frac{5\pi}{3}\right) = \arcsin\left(-\frac{\sqrt{3}}{2}\right) = \,? = -\frac{\pi}{3}$



$\sin\,? = -\frac{\sqrt{3}}{2}$

$\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\arcsin\left(\sin\frac{5\pi}{3}\right) = \arcsin\left(\sin\left(-\frac{\pi}{3}\right)\right) = -\pi/3$

c. $\cos(\cos^{-1}\pi)$ = undefined.

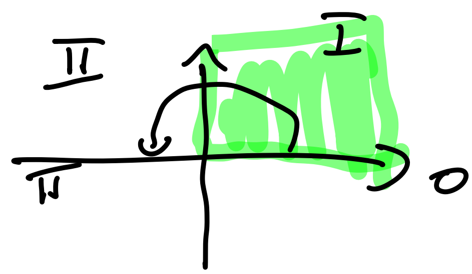
D: for $\cos^{-1}$ $[-1, 1]$ π is not $[-1, 1]$

When we compose trig functions with other trig inverses (i.e. tangent composed with arccosine) we can make use of right triangles to find exact values of compositions that involve inverse trig functions.

Example 3. *Find the exact value.*

a. $\tan\left(\arccos \frac{2}{3}\right)$

b. $\cos\left[\arcsin\left(-\frac{3}{5}\right)\right]$

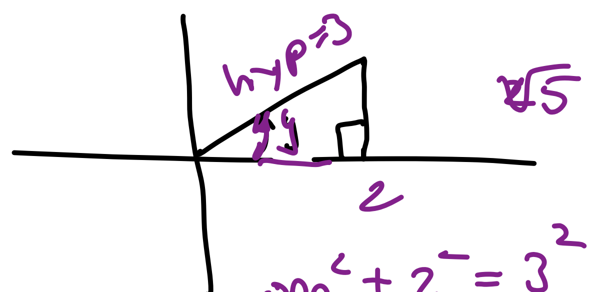


a) $\tan\left(\arccos \frac{2}{3}\right) = \tan y = \frac{\sqrt{5}}{2}$

$\arccos \frac{2}{3} = y$

$\cos y = \frac{2}{3}$

$\frac{adj}{hyp} = \frac{2}{3}$



$opp^2 + 2^2 = 3^2$

$opp^2 = 9 - 4$

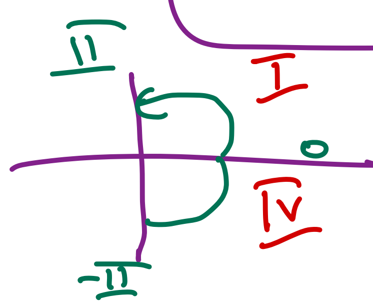
$opp^2 = 5$

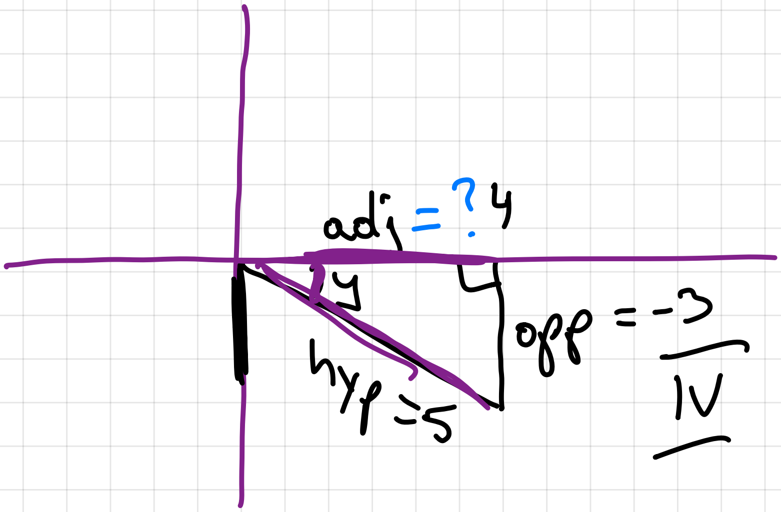
$opp = \sqrt{5}$

b) $\cos\left(\arcsin\left(-\frac{3}{5}\right)\right) = \cos y = \frac{4}{5}$

$\arcsin\left(-\frac{3}{5}\right) = y$

$\sin y = -\frac{3}{5}$





$$\sin y = -\frac{3}{5}$$

$$\frac{opp}{hyp} = \frac{-3}{5}$$

$$adj^2 + (-3)^2 = 5^2$$

$$adj^2 = 16$$

$$adj = 4$$

Examples skills needed in Calculus

Example 4. *Write each of the following as an algebraic expression in x.*

a. $\sin(\arccos 3x), \quad 0 \leq x \leq \frac{1}{3}$

b. $\cot(\arccos 3x), \quad 0 \leq x < \frac{1}{3}$

next time