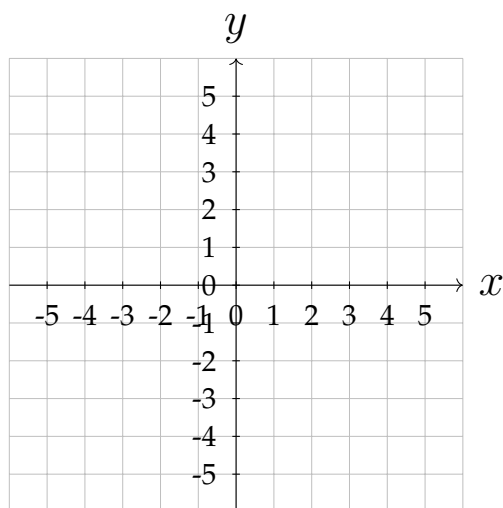


## 1. Circles

Next, we will start graphing specific categories of equations such as circles, ellipses, hyperbolas and parabola. We begin with circles.

**Definition:** A circle is the set of all points in a plane that are equidistant from a fixed point called \_\_\_\_\_. The fixed distance from any point on the circle to the center is called the \_\_\_\_\_.



### Standard form of an equation of a circle

Given a circle centered at  $(h, k)$  with radius  $r$ , the standard form is

### Example 1

1.  $(x - 4)^2 + (y + 3)^2 = 25$

2.  $x^2 + (y - 1/2)^2 = 12$

3.  $x^2 + y^2 = 7$

**Example 2** Write the standard form of an equation of a circle with center  $(-4, 6)$  and radius 2.

## 2. Recall: Distance and Midpoint Formulas

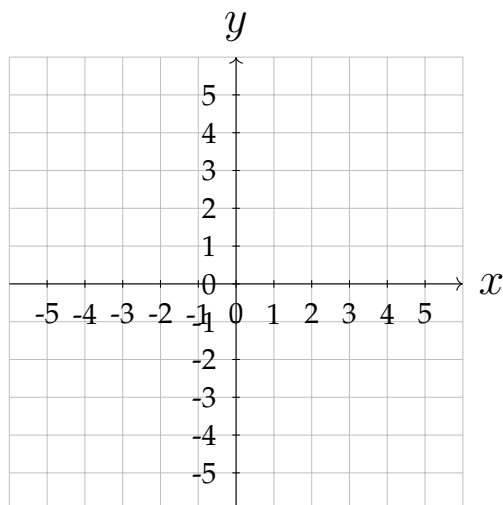
**Distance Formula:** The distance  $d$  between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Midpoint Formula:** The midpoint  $M(x, y)$  of a line segment with endpoints  $(x_1, y_1)$  and  $(x_2, y_2)$  is:

$$M \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**Example 3** Write the standard form of an equation of a circle with the endpoints of the diameter as  $(-1, 0)$  and  $(3, 4)$ . Graph the circle.



### General form of an Equation of a circle

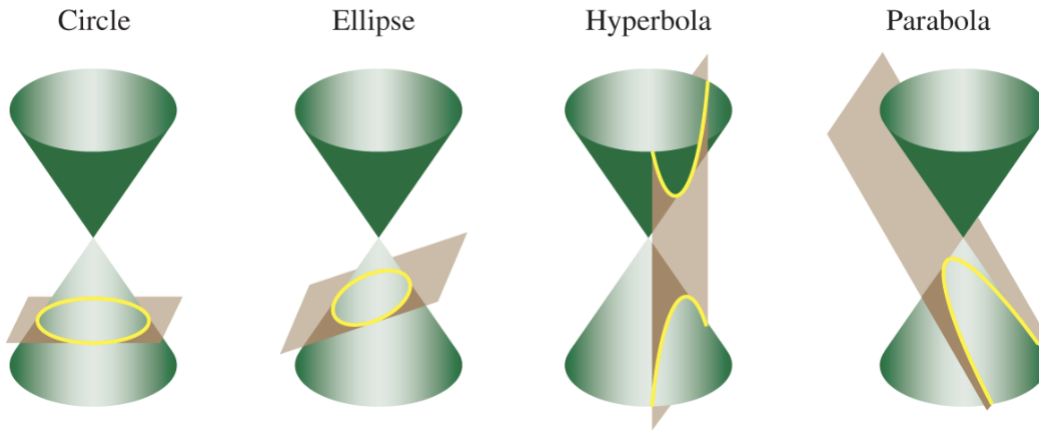
An equation of a circle written in the form  $x^2 + y^2 + Ax + By + C = 0$  is called the general form of an equation of a circle.

**Example 4** Write the equation of the circle in standard form by **completing the square**.

$$x^2 + y^2 + 10x - 6y + 25 = 0$$

## 2. Ellipses

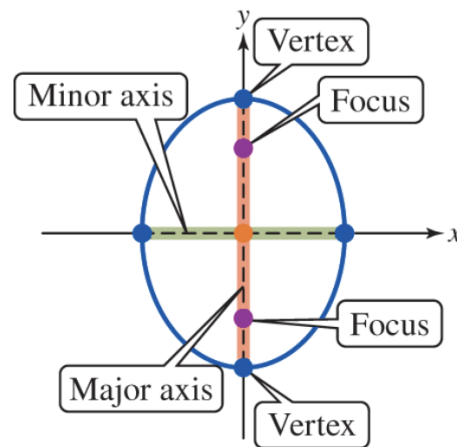
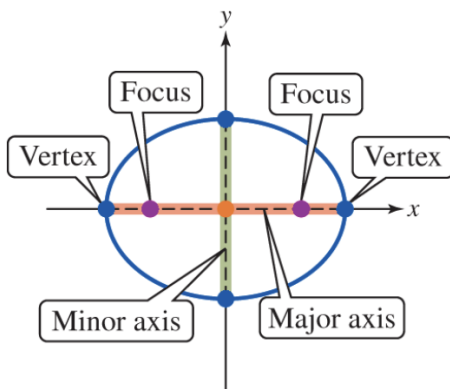
In this section we study ellipses from a geometric point of view.



**Definition:** An ellipse is the set of all points in the plane such that the sum of the distance between a point  $(x, y)$  and two fixed points is a constant.

The fixed points are called

An ellipse may be elongated in any direction. We will study only those that are elongated horizontally and vertically.



### 3. Equations and Graphs of ellipses

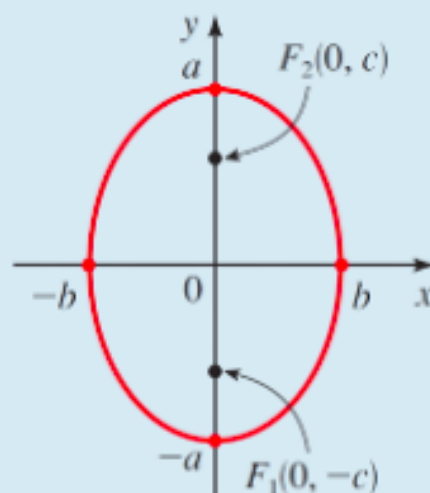
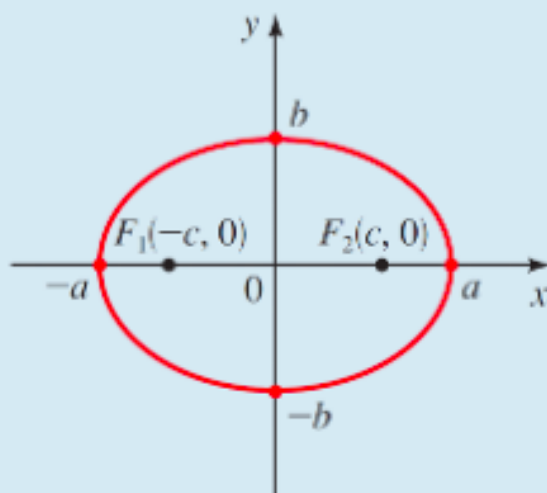
The following box summarizes the equation and features of an ellipse centered at the origin.

#### ELLIPSE WITH CENTER AT THE ORIGIN

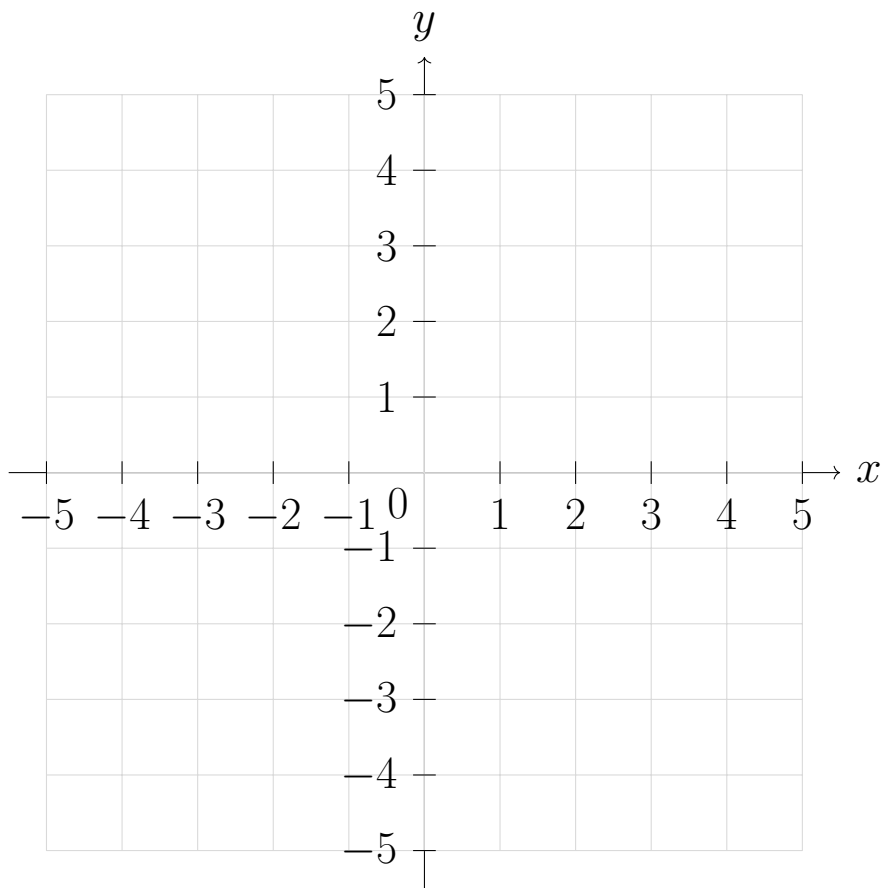
The graph of each of the following equations is an ellipse with center at the origin and having the given properties.

|                   |                                                        |                                                        |
|-------------------|--------------------------------------------------------|--------------------------------------------------------|
| <b>EQUATION</b>   | $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$<br>$a > b > 0$ | $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$<br>$a > b > 0$ |
| <b>VERTICES</b>   | $(\pm a, 0)$                                           | $(0, \pm a)$                                           |
| <b>MAJOR AXIS</b> | Horizontal, length $2a$                                | Vertical, length $2a$                                  |
| <b>MINOR AXIS</b> | Vertical, length $2b$                                  | Horizontal, length $2b$                                |
| <b>FOCI</b>       | $(\pm c, 0)$ , $c^2 = a^2 - b^2$                       | $(0, \pm c)$ , $c^2 = a^2 - b^2$                       |

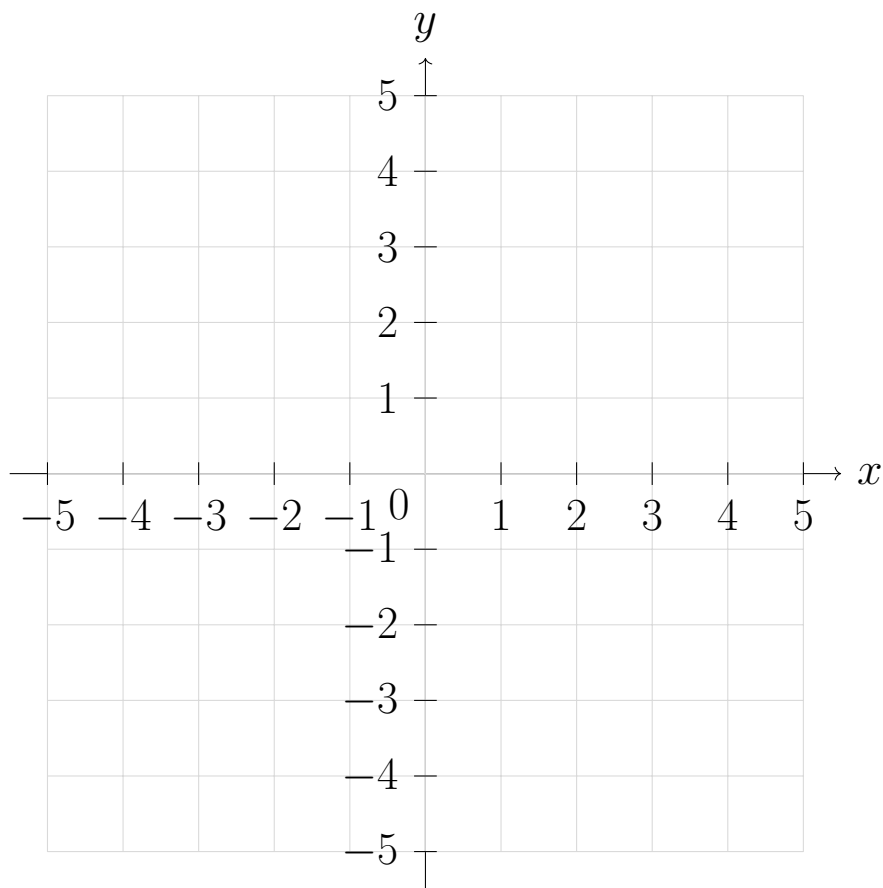
**GRAPH**



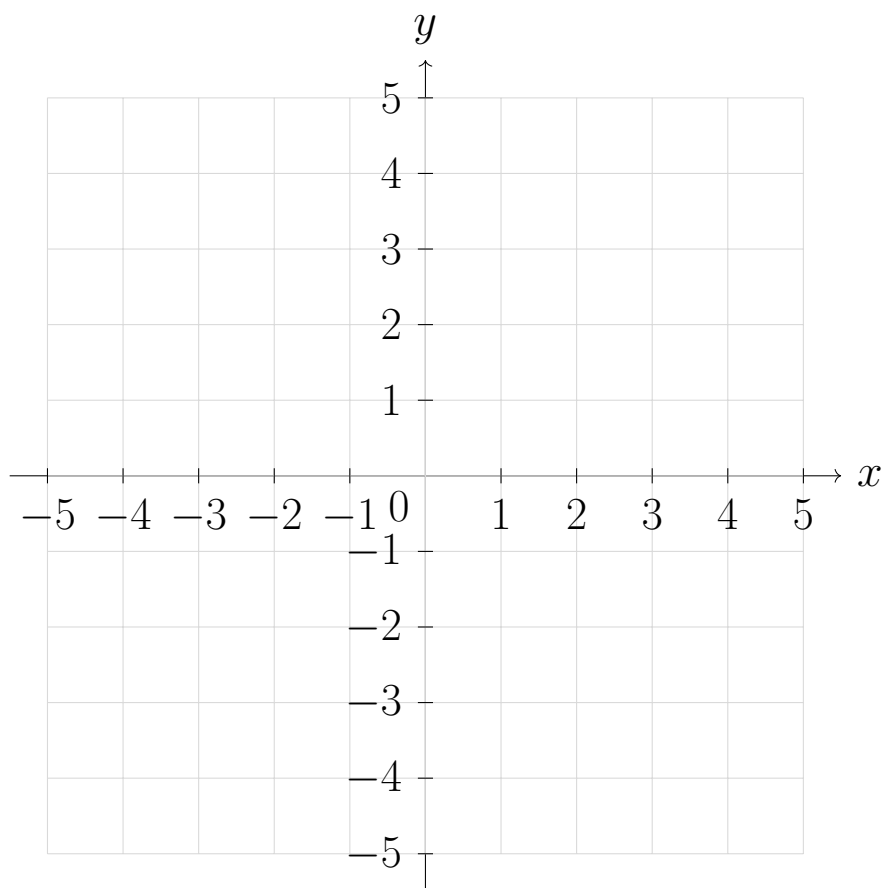
**Example 5:** Given  $\frac{x^2}{16} + \frac{y^2}{9} = 1$ , graph the ellipse and identify the center, foci and vertices.



**Example 6:** Find the foci of the ellipse  $25x^2 + 9y^2 = 225$ , and sketch its graph.



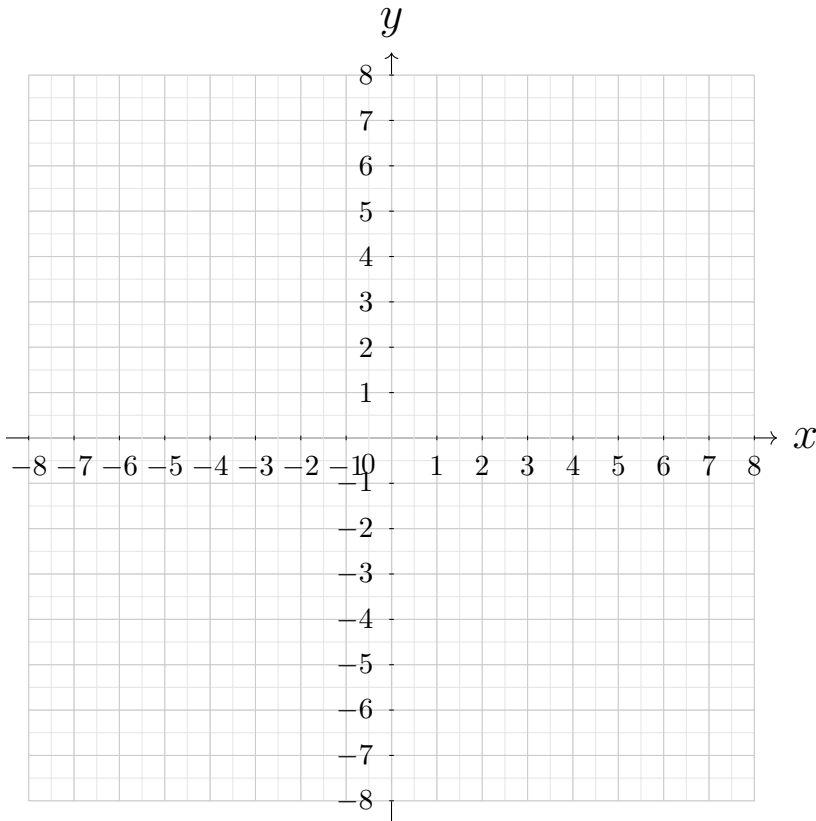
**Example 7:** The vertices of an ellipse are  $(\pm 4, 0)$ , and the foci are  $(\pm 2, 0)$ . Find its equation, and sketch the graph.



#### 4. Graphing and Ellipse centered at $(h, k)$

Replacing  $x$  by  $x - h$  and  $y$  by  $y - k$  shifts the graph of an ellipse  $h$  units horizontally and  $k$  units vertically.

**Example 8:** Given  $\frac{(x - 3)^2}{16} + \frac{(y + 2)^2}{25} = 1$ , graph the ellipse and identify the center, foci and vertices.



**Example 9: Practice:** Given  $3x^2 + 7y^2 + 6x - 28y + 10 = 0$ , write the equation of the ellipse in standard form, graph the ellipse and identify the center, foci and vertices.

