Math 140: Tuesday, Oct 22, Day 15 Notes

Agenda for Tuesday, Oct 22, 2024

Chapter 3 Probability **1**

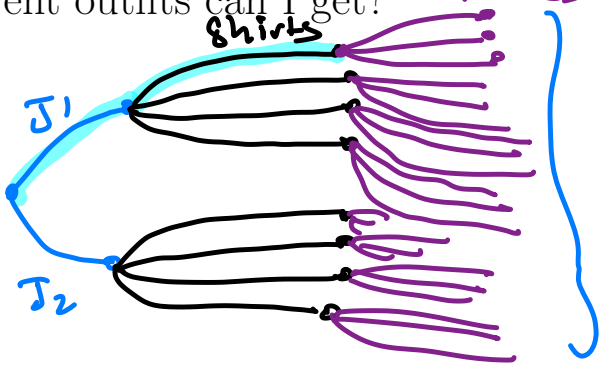
Coming up

! Online HW 9 will be posted this evening

- Office Hours : 2:30 - 3:45 pm C162 today

Counting Methods

Example 1. If I have 2 jackets, 4 shirts and 3 pants how many different outfits can I get?



⟹ 24 different outfit

$2 \cdot 4 \cdot 3 = 24$

Fundamental principle of counting If a first experiment can be performed in M distinct ways, and a second experiment can be performed in N distinct ways, then the two experiments can be performed in

M · N distinct ways.

digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 → 10 digits

Example 2. How many 4 digit PIN's are possible when the first digit can't be repeated?

10 · 9 · 9 · 9

10 ↓ choice

# of distinct 4 digit PINs:
$10 \cdot 9 \cdot 9 \cdot 9 = 7290$

Example 3. A telephone number has 10 digits:

(area code) — — — — — — — —

1. How many possible telephone numbers are there in each area code?

(area code) 10 10 10 10 10 10 10

$10^7 = 10{,}000{,}000$ phone numbers

2. How many telephone numbers are there in each area code where the first digit can't begin with a 0 and the last four digits can't repeat?

(area code) 9 10 10 10 9 8 7

Ans: 4,536,000

Example 4. How many ways can eight people line up for a photograph?

8 7 6 5 4 3 2 1

$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 8! = 40{,}320$

8 → Math → PROB → !

calculator

Factorial Recall: $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

Example 5. A local television station has eleven slots for commercials during a special broadcast. Six restaurants and 5 stores have bought slots for the broadcast.

(a) In how many ways can the commercials be arranged?

11 10 9 8 7 — — — — — 2 1

$11! = 39$ mill

(b) In how many ways can the commercials be arranged so that the restaurants are grouped together and the stores are grouped together?

6 5 4 3 2 1 (rest) 5 4 3 2 1 (stores)

$6! \cdot 5!$

5 4 3 2 1 (stores) or 6 5 4 3 2 1 (rest)

$5! \cdot 6!$

Ans $6! \cdot 5! + 5! \cdot 6! = 172,800$

In English we use the word "combination" loosely. We say things like:

- "My fruit salad is a combination of apples, grapes and bananas"

order does not matter

AGB = ABG

- "The combination to the safe is 472".

472 ≠ 427 order matter.

When the order doesn't matter, it is a **Combination**.

When the order does matter it is a **Permutation**.

Permutations and Combinations: Consider having the letters A, B, C, D. Let's choose two of the letters.

permutation

In how many different ways can we arrange two of the 4 letters? In how many different ways can we combine two of the 4 letters?

combs

AB BA CA DA

AC BC CB DB

AD DD CD DC

combinations = 6

permutation = 12

Permutations: We use permutations when n is the number of things to choose from, and we choose r of them, no repetitions, order matters.

Formula: $_nP_r = \frac{n!}{(n-r)!}$

ex: $n=4$ letters
$r=2$

$_4P_2 = \frac{4!}{(4-2)!} = \frac{4\cdot3\cdot2\cdot1}{2\cdot1} = 12$

Calculator instructions:

n → Math → PROB → nPr (#2) → enter r

Example 6. A volleyball team has 10 players, how many 6 player starting line up are possible?

$n=10$
$r=6$
no repetion: a player can't be two positions
order matters

$_{10}P_6 = 151{,}200$

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extra

26 letters
2 at a time

$_{26}P_2 = 650$

Combinations We use combination when n is the number of things to choose from, and we choose r of them, no repetition, order doesn't matter.

Formula: $nCr = \frac{n!}{(n-r)!\,r!}$

$n=4$ letters
$r=2$

$4C_2 = \frac{4!}{(4-2)!\,2!} = \frac{4\cdot 3\cdot 2\cdot 1}{2\cdot 1\cdot 2\cdot 1} = 6$

Calculator instructions:

nCr Math → PRB → nCr → r

Example 7. Four lawyers are to be selected from a group of 30 lawyers to work on a project.

(a) How many different ways can the lawyers be selected to work on the project?

$n=30$
$r=4$

$30C_4 = 27{,}405$

(b) In how many ways can the group of 4 be selected if a certain lawyer must work on the project?

$29C_3 = 3{,}654$

because 1 is already

Example 8. In a student club with 9 sophomores and 11 freshman members, a 5- member committee will be randomly chosen. How many ways can a committee be chosen with 3 sophomores and 2 freshmen?

$9C_3$ → 84 # of ways to choose soph

$11C_2$ → 55 # of ways to choose fresh

$84 \cdot 55 = 4620$

Combinations We use combination when n is the number of things to choose from, and we choose r of them, no repetition, order doesn't matter.

Formula:

Calculator instructions:

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