

Weighted mean:

Example 1. Your grade in this class is calculated as weighted mean.

Grade Distribution
3 In Class Exams 45%
Online Canvas Homework 20%
Excel Assignments 10%
In class group activities 10%
Final exam 15%

Monday, Sept 24th

Section 3.2 Measures of variation

In some data sets, the data values are concentrated closely near the mean; in other data sets, the data values are more widely spread out from the mean. We will discuss three measures of variation: the **range**, the standard deviation, and the variance.

Range: = highest value - lowest value

The higher the range, the more variation in the data.

Standard Deviation: is an average deviation from the mean of a data set.

σ - st. dev. of the population

↓
sigma

s - st. dev. of a sample

Stats section 1 exam scores	Stats section 2 exam scores
49.5, 57, 66, 73, 78, 85, 92	67, 68.5, 71, 72, 74.5, 75
$\bar{x} = 71.5$	$\bar{x} = 71.3$
$\sigma = 14$	$\sigma = 2.9$
$s = 15.1$	$s = 3.2$

Variance: $= (\text{st. dev})^2 \rightarrow \underline{\underline{\text{no units}}}$

first data set

$$\sigma^2 = \text{population variance} = (14)^2 = 296$$

$$s^2 = \text{sample variance} = (15.1)^2 = 228$$

second data set

$$\sigma^2 = (2.9)^2 = 8.4$$

$$s^2 = (3.2)^2 = 10.4$$

Standard Deviation of Grouped Frequency Distribution

<u>MPG</u>	Frequency
24 - 31	12
32 - 39	15
40 - 47	8
48 - 55	4
56 - 63	1

L_1 : Midpoints of the classes
 $\frac{24+31}{2} = 27.5, \dots$

L_2 : Frequency

1 var stats L_1, L_2

$S = 8.5$ mpg
in calculator S_x

Agenda for Monday, Sep 23rd 2024

Section 3.3 Measures of Position	1
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Coming up

- ! HW 8 will be posted this evening
- Second **Excel Homework** will be posted Wednesday

Section 3.3 Measures of position

The standard deviation is useful when comparing data values that come from different data sets. If the data sets have different means and standard deviations, then comparing the data values directly can be misleading.

Let x be a data value. A **z-score** corresponding to x tells you how many st.dev. away from the mean the data value x is.

Formula :

$$z_{\text{value}} = \frac{\text{value} - \text{mean}}{\text{st.dev.}}$$
$$z_x = \frac{x - \bar{x}}{s}$$

Ex $\bar{x} = 85$
 $s = 10$

$$z_{95} = \frac{95 - 85}{10} = \frac{10}{10} = 1$$

positive z score represent value above the mean

$$z_{70} = \frac{70 - 85}{10} = \frac{-15}{10} = -1.5$$

negative z-scores represent values below mean

Example 1. The average test score for Mr. Jone's class is 124 with a standard deviation of 16. The average test score for Ms. Siegel's class is 72 with a standard deviation of 8. Tina and Melanie were comparing themselves to see who did better on their prospective exams. Tina scored a 118 in Mr. Jone's exam while Melanie scored a 67 on Ms. Siegel's exam. Find the z-scores and compare.

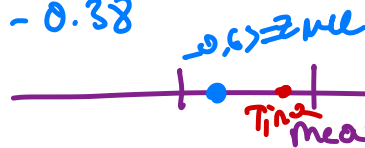
J

$$\bar{x} = 124$$

$$s = 16$$

Tina: 118

$$z_{118} = \frac{118 - 124}{16} = -0.38$$



S

$$\bar{x} = 72$$

$$s = 8$$

Mel: 67

$$z_{\text{Mel}} = \frac{67 - 72}{8} = -0.63$$

Tina scored better

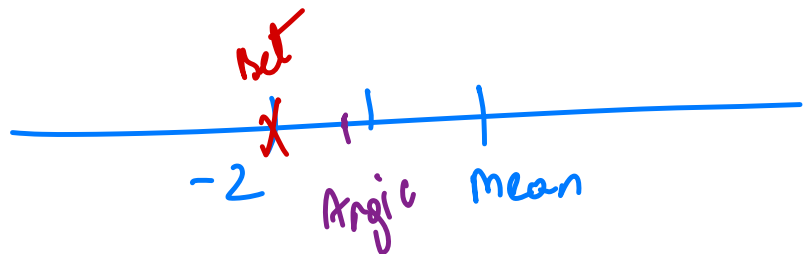
Practice: Two swimmers, Angie and Beth, from different teams, wanted to find out who had the fastest time for the 50 meter freestyle when compared to her team. The table below shows their times.

Swimmer	Time in Seconds	Team Mean Time	Team Standard Deviation
Angie	26.2	27.2	0.8
Beth	27.3	30.1	1.4

Which swimmer had the fastest time when compared to her team?

$$z_{\text{Angie}} = -1.25$$

$$z_{\text{Beth}} = -2$$



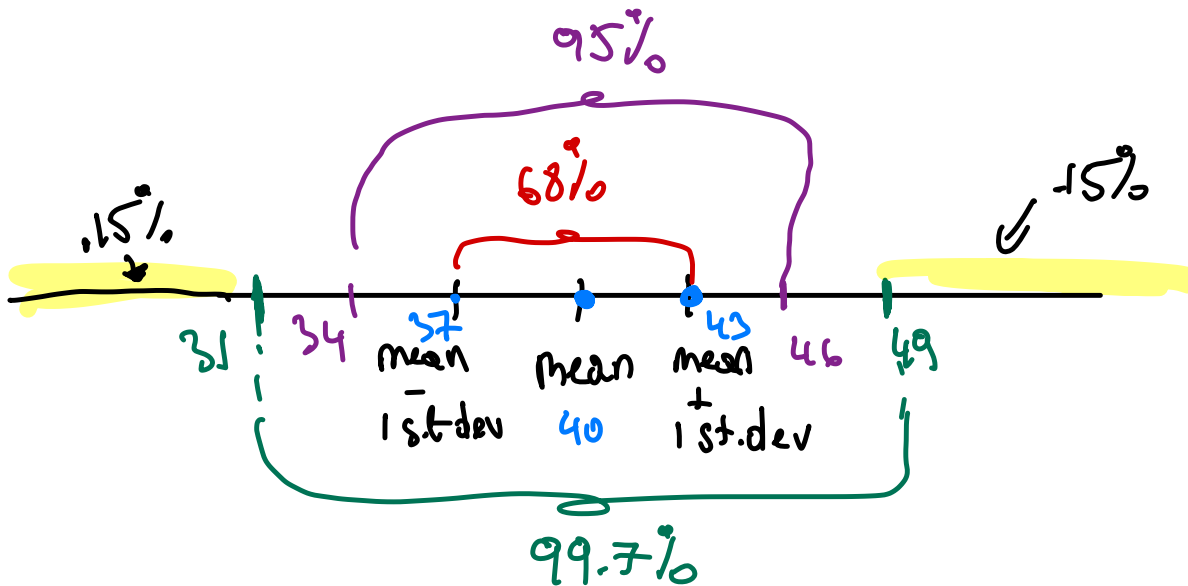
Beth was faster compared to her team

Empirical Rule

For data having a distribution that is bell-shaped and symmetric:

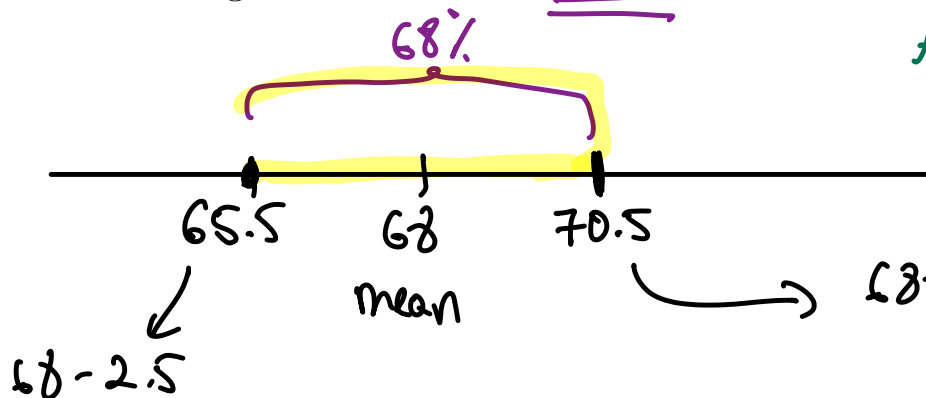
- 68% of the data lies within one standard deviation of the mean
- 95% of the data lies within **two** standard deviation of the mean
- 99.7% of the data lies within three standard deviation of the mean

Visual of Empirical Rule, use mean = 40 and standard deviation = 3



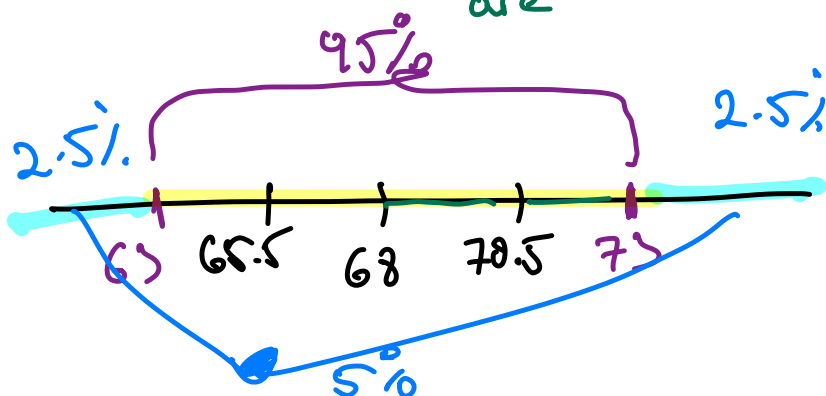
Example 2. The average height of adult men in the US is 68 inches with a standard deviation of 2.5 inches. Using Empirical Rule 71 inches

1. Find the heights of the middle 68% of adult men.



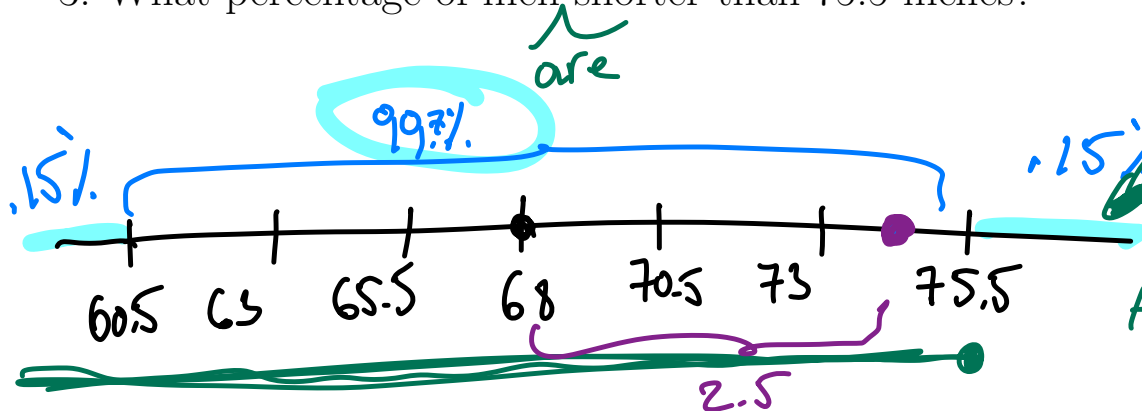
Ans: 68% of men in US have a height between 65.5 and 70.5 inches.

2. What percentage of men are taller than 73 inches?



Ans: 2.5% of men in US are taller than 73 inches.

3. What percentage of men are shorter than 75.5 inches?



Ans: .15% + 99.7% = 99.85%

4. What is your height if your z-score is 2.5?

$$z = \frac{x - \bar{x}}{s}$$

$$2.5 = \frac{x - 68}{2.5}$$

=> solve for x = ?

Ans: x = 74.25