Math 140: Thursday, Oct 10th, Day 14 Notes

Agenda for Thursday, Oct 10th, 2024	
Chapter 3 Probability	**1**

Coming up

- ! Online HW 7 due Today
- ! Written HW 4 due Tuesday
- Office Hour **tomorrow 12-1:15pm** in C162 (not today at 1pm)
- **Exam 2** on Thursday Oct 17th - Study guide and Practice Exam posted by end of Friday
- Carrera de los Lancers - 5k this Saturday; meet in front of gym at 10am

Recall: See probability flow chart

Probabilities involving "AND" continued

$P(* | \star) = \frac{P(* \text{ and } \star)}{P(\star)}$

Independent events revisited:

1. Consider drawing two cards (one at a time) from a deck of cards **with replacement**. Let K_{1st} be the event of "drawing a King the first time" and K_{2nd} be the event of "drawing a King the second time". Find:

   $P(K_{2nd}) = \frac{4}{52}$

   $P(K_{2nd}|K_{1st}) \overset{*}{=} \frac{P(\overbrace{K_{2nd} \text{ and } K_{1st}}^{\text{independent}})}{P(K_{1st})} = \frac{P(K_{2nd}) \cdot \cancel{P(K_{1st})}}{\cancel{P(K_{1st})}} = P(K_{2nd})$

   $= \frac{4}{52}$

   $P(A|B) = P(A)$ if A and B independent

   Event A and B are independent if and only if:

   - $P(A \text{ and } B) = P(A) \cdot P(B)$
   - $P(A|B) = P(A)$
   - $P(*|\star) = P(*)$

2. Consider drawing two cards (one at a time) from a deck **without replacement**. Let K_{1st} be the event of "drawing a King the first time" and K_{2nd} be the event of "drawing a King the second time". Find:

   $P(K_{2nd}|K_{1st}) = \frac{3}{51}$

   can't break this down (events are not independent)

   $P(K_{2nd}|K_{1st}) = \frac{P(K_{2nd} \text{ and } K_{1st})}{P(K_{1st})}$

   $\boxed{P(K_{2nd} \text{ and } K_{1st}) = P(K_{1st}) \cdot P(K_{2nd}|K_{1st})} = \frac{4}{52} \cdot \frac{3}{51}$

General Multiplication Rule

$P(* \text{ and } \star) = P(*) \cdot P(\star | *)$

$*, \star$ not independent

The probability that two events A and B will occur in sequence is

$$\boxed{P(A \text{ and } B) = P(A) \cdot P(B|A).}$$

If events A and B are independent, then the rule can be simplified to

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

$P(B|A) = B$

Example 1. In a survey, 510 adults were asked if they drive a pickup truck and if they drive a Ford. The results showed that one in six adults surveyed drives a pickup truck, and three in ten adults surveyed drive a Ford. Of the adults surveyed that drive Fords, two in nine drive a pickup truck.

T

$P(T) = \frac{1}{6}$, $P(F) = \frac{3}{10}$

1. Find the probability that a randomly selected adult drives a pickup truck, **given** that the adult drives a Ford.

$\boxed{P(T|F) = \frac{2}{9}}$ $= 0.222$

2. Are the events "driving a Ford" and "driving a pickup truck" independent or dependent? Explain.

$P(T|F) \stackrel{?}{=} P(T)$

$\frac{2}{9} \stackrel{?}{=} \frac{1}{6}$ No so not independent

3. Find the probability that a randomly selected adult drives a Ford pick up truck.

truck and Ford

$P(T \text{ and } F) = P(T) \cdot P(T|F)$

$= \frac{1}{6} \cdot \frac{2}{9} = \frac{1}{27} \approx .037$

Using Tree Diagrams

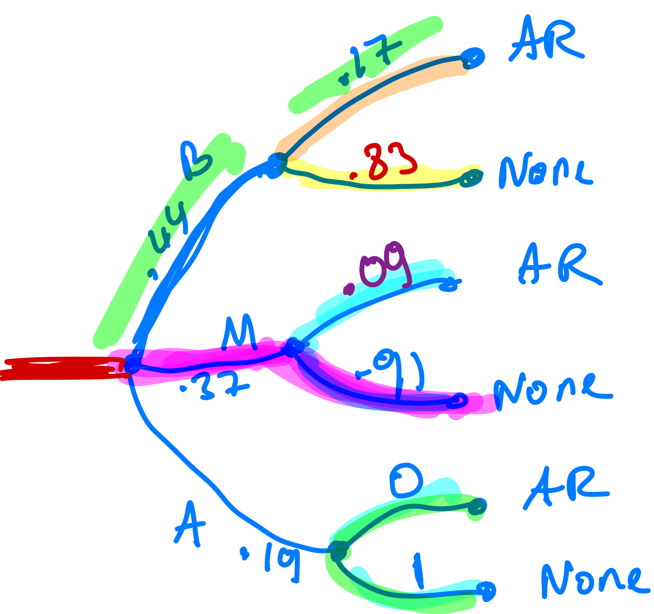
$P(B) = 0.44$, $P(M) = .37$, $P(A) = .19$
binge, mod, abstain

The kind of picture that helps us look at conditional probabilities and multiplication rule for dependent events is called a **tree diagram**.

Example 2. According to a study 44% of college students engage in binge drinking, 37% drink moderately, and 19% abstain entirely. Another study finds that among binge drinkers, 17% have been involved in an alcohol-related automobile accident, while among non-bingers of the same age, only 9% have been involved in such accidents.

AR

1. Build a tree diagram with corresponding probabilities on each possible outcome.



$P(AR \mid B) = .17$

$P(None \mid B) = .83$

$P(AR \mid M) = .09$

$P(None \mid M) =$

2. What's the probability that a randomly selected college student will be a binge drinker **and** has had an alcohol-related car accident?

$P(B \text{ and } AR) = P(B) \cdot P(AR \mid B) = (.44)(.17) = .075$

3. What's the probability that a randomly selected college student will have an alcohol-related car accident **given** they are a moderate drinker?

AR M

$P(AR \mid M) = .09$

4. What's the probability that a randomly selected college student will be a moderate drinker **and** has had no alcohol-related car accident?

$P(M \text{ and } None) = (0.37) \cdot (0.91) =$