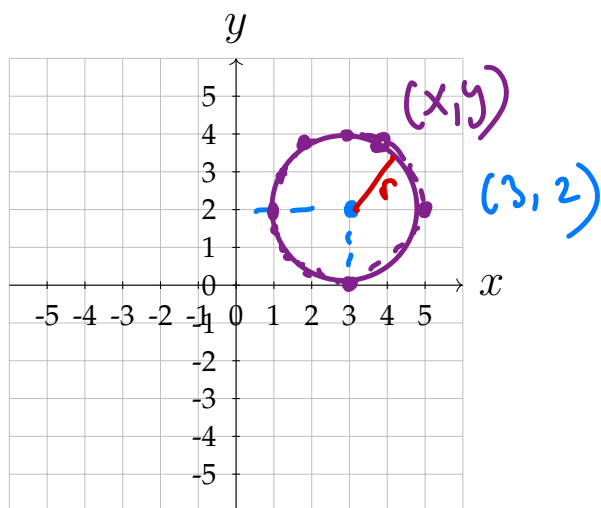


1. Circles

Next, we will start graphing specific categories of equations such as circles, ellipses, hyperbolas and parabola. We begin with circles.

Definition: A circle is the set of all points in a plane that are equidistant from a fixed point called center. The fixed distance from any point on the circle to the center is called the radius.



r - radius
 $r = 2$
 center: (h, k)
 $\downarrow$
 x coordinate of center
 $\rightarrow$
 y coord of center

Standard form of an equation of a circle

Given a circle centered at (h, k) with radius r , the standard form is

$$(x - h)^2 + (y - k)^2 = r^2$$

$\downarrow$
 standard formula for a
 circle

Example 1

$$1. (x-4)^2 + (y+3)^2 = 25$$

$$(x-4)^2 + (y-(-3))^2 = 5^2$$

center: $(4, -3)$
radius: $r=5$

$$2. x^2 + (y-1/2)^2 = 12$$

$$(x-0)^2 + (y-1/2)^2 = (\sqrt{12})^2$$

center: $(0, 1/2)$

radius: $r = \sqrt{12} = \sqrt{4 \cdot 3}$
 $r = \sqrt{4} \sqrt{3} = 2\sqrt{3}$

$$3. x^2 + y^2 = 7$$

$$(x-0)^2 + (y-0)^2 = (\sqrt{7})^2$$

$(0, 0)$
 $r = \sqrt{7}$

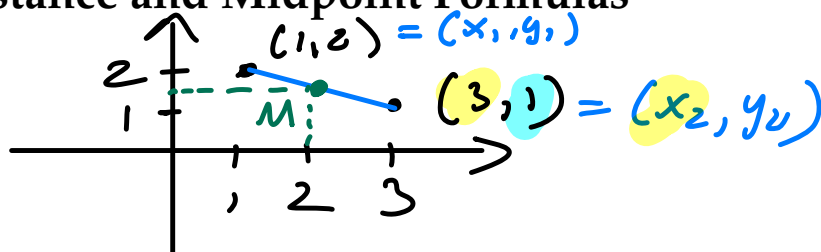
Example 2 Write the standard form of an equation of a circle with center $(-4, 6)$ and radius 2.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-(-4))^2 + (y-6)^2 = 2^2$$

$$(x+4)^2 + (y-6)^2 = 4$$

2. Recall: Distance and Midpoint Formulas



Distance Formula: The distance d between two points (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(3-1)^2 + (1-2)^2} = \sqrt{4+1} = \sqrt{5}$$

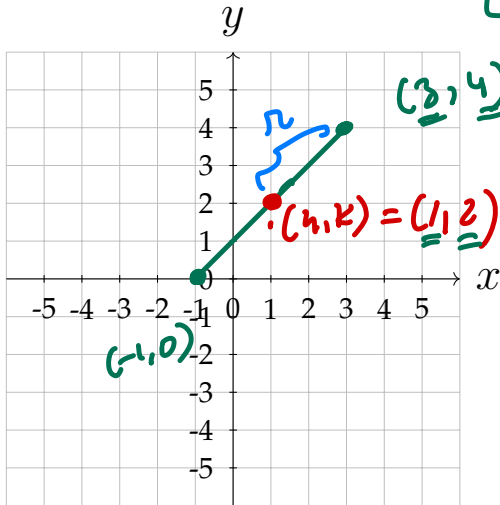
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula: The midpoint $M(x, y)$ of a line segment with endpoints (x_1, y_1) and (x_2, y_2) is:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M\left(\frac{1+3}{2}, \frac{2+1}{2}\right) = \left(\frac{4}{2}, \frac{3}{2}\right) = (2, 1.5)$$

Example 3 Write the standard form of an equation of a circle with the endpoints of the diameter as $(-1, 0)$ and $(3, 4)$. Graph the circle.



twice the radius

$$(x - \underline{h})^2 + (y - \underline{k})^2 = r^2$$

$$(h, k) = M\left(\frac{-1+3}{2}, \frac{0+4}{2}\right) = (1, 2)$$

$$(x - 1)^2 + (y - 2)^2 = (\sqrt{8})^2$$

$$r = \sqrt{(3-1)^2 + (4-2)^2} = \sqrt{4+4} = \sqrt{8} = \sqrt{4 \cdot 2}$$

$$r = 2\sqrt{2}$$

$$(x - 1)^2 + (y - 2)^2 = 8$$

General form of an Equation of a circle

An equation of a circle written in the form $x^2 + y^2 + Ax + By + C = 0$ is called the general form of an equation of a circle.

Example 4 Write the equation of the circle in standard form by completing the square.

$$x^2 + y^2 + 10x - 6y + 25 = 0$$

general form

$$(x - h)^2 + (y - k)^2 = r^2$$

standard form

Steps for completing the square

$$1) \underbrace{x^2 + 10x}_{\text{binomial in } x} + \underbrace{y^2 - 6y}_{\text{binomial in } y} = -25$$

$$2) \underbrace{x^2 + 10x + 25}_{\text{take half of coeff of } x, \text{ square it and add to both sides}} + \underbrace{y^2 - 6y + 9}_{\text{take half of coeff of } y, \text{ square it and add to both sides}} = -25 + 25 + 9$$

take half of coeff of x , square it and add to both sides

$$\left(\frac{10}{2}\right)^2 = 5^2 = 25$$

$$\left(-\frac{6}{2}\right)^2 = (-3)^2 = 9$$

$$3) (x + 5)^2 + (y - 3)^2 = 9$$

- ① Rewrite the general equation by grouping the x terms together and y terms together
- ② Complete the square for binomial in x and for binomial in y .