

3. Equations and Graphs of ellipses

The following box summarizes the equation and features of an ellipse centered at the origin.

ELLIPSE WITH CENTER AT THE ORIGIN

The graph of each of the following equations is an ellipse with center at the origin and having the given properties.

EQUATION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a > b > 0$$

VERTICES

$$(\pm a, 0)$$

MAJOR AXIS

Horizontal, length $2a$

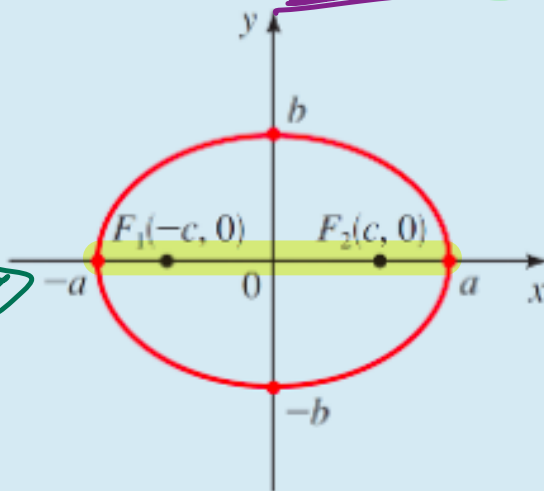
MINOR AXIS

Vertical, length $2b$

FOCI

$$(\pm c, 0), \quad c^2 = a^2 - b^2$$

GRAPH



bigger number with the x term

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

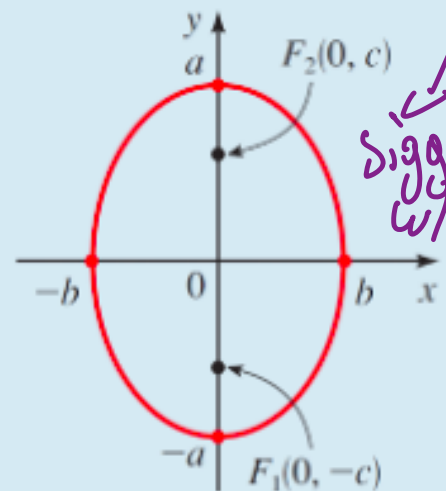
$$a > b > 0$$

$$(0, \pm a)$$

Vertical, length $2a$

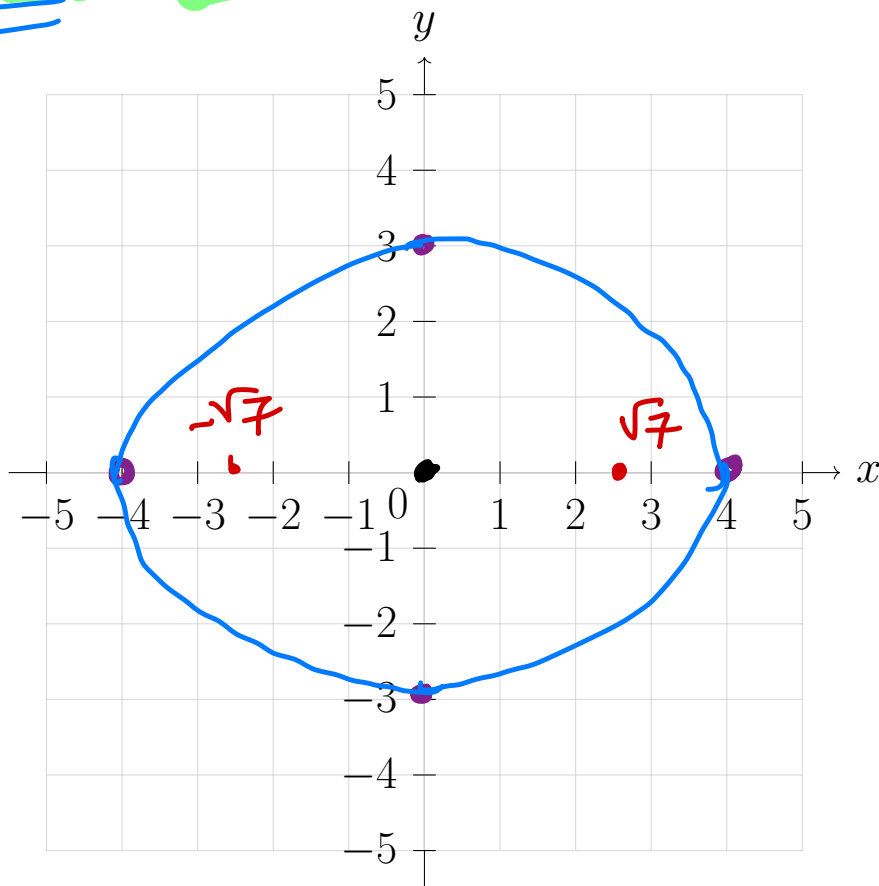
Horizontal, length $2b$

$$(0, \pm c), \quad c^2 = a^2 - b^2$$



bigger w/ y

Example 5: Given $\frac{x^2}{16} + \frac{y^2}{9} = 1$, graph the ellipse and identify the center, foci and vertices.



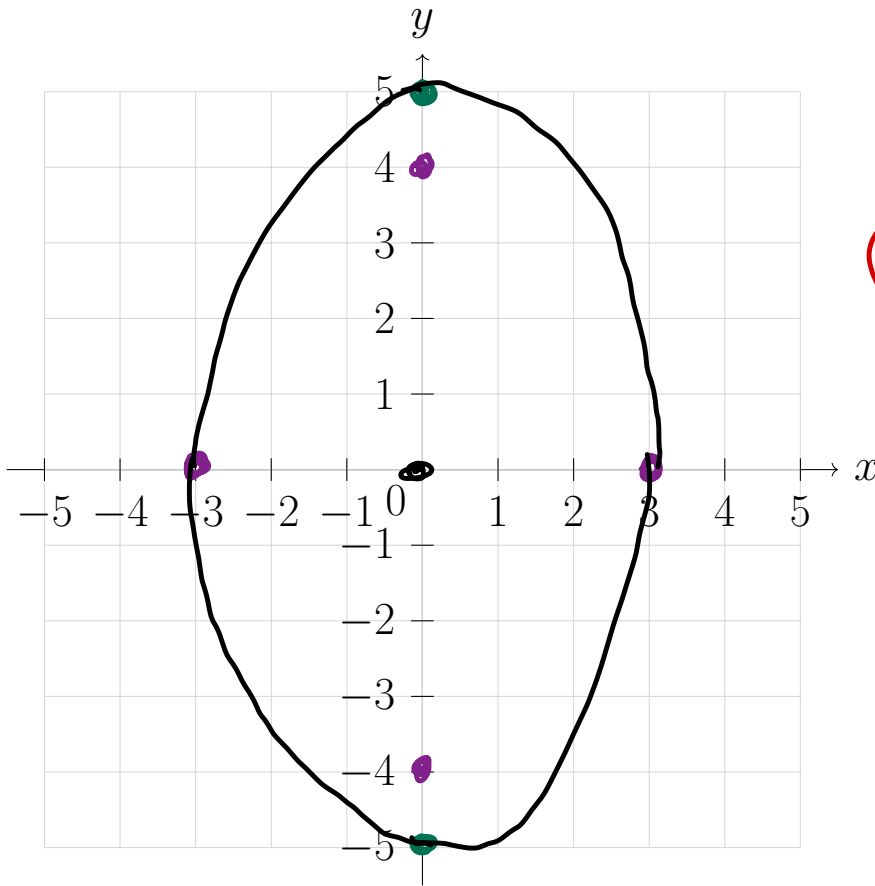
- elongated x axis
- foci :
- vertex : $(4, 0)$ $(-4, 0)$
- foci : $(\sqrt{7}, 0)$ $(-\sqrt{7}, 0)$
- $c^2 = a^2 - b^2$
- $= 16 - 9 = 7$
- $c = \sqrt{7}$

$$\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1 \quad \text{compare} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a = 4 \rightarrow \text{vertex}$$

$$b = 3 \rightarrow \text{covertex}$$

Example 6: Find the foci of the ellipse $25x^2 + 9y^2 = 225$, and sketch its graph.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

or

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

divide by 225

$$\frac{25x^2}{225} + \frac{9y^2}{225} = \frac{225}{225}$$

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

$$\frac{x^2}{3^2} + \frac{y^2}{5^2} = 1$$

elongated along y axis

- vertices: $(0, a)$ & $(0, -a) : (0, 5), (0, -5)$

- foci:

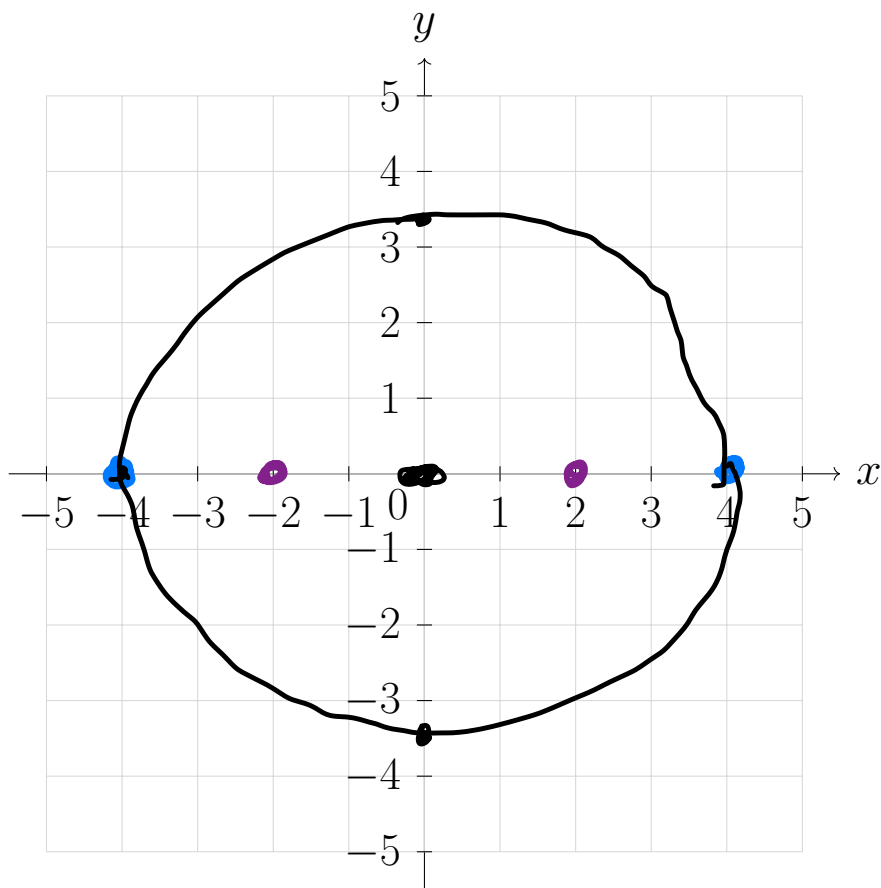
$$c^2 = a^2 - b^2$$

$$c^2 = 5^2 - 3^2 = 25 - 9 = 16$$

$$c = 4$$

- co-vertices: $(b, 0)$ & $(-b, 0)$
 $(3, 0)$ & $(-3, 0)$

Example 7: The vertices of an ellipse are $(\pm 4, 0)$, and the foci are $(\pm 2, 0)$. Find its equation, and sketch the graph.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a = 4$$

$$c = 2$$

$$c^2 = a^2 - b^2$$

$$4 = 16 - b^2$$

$$-12 = -b^2$$

$$b^2 = 12$$

$$b = \sqrt{12} \approx 3.46$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{4^2} + \frac{y^2}{(\sqrt{12})^2} = 1$$

$$\frac{x^2}{16} + \frac{y^2}{12} = 1$$

Calculator instructions

1) solve for y

$$\frac{y^2}{12} = 1 - \frac{x^2}{16}$$

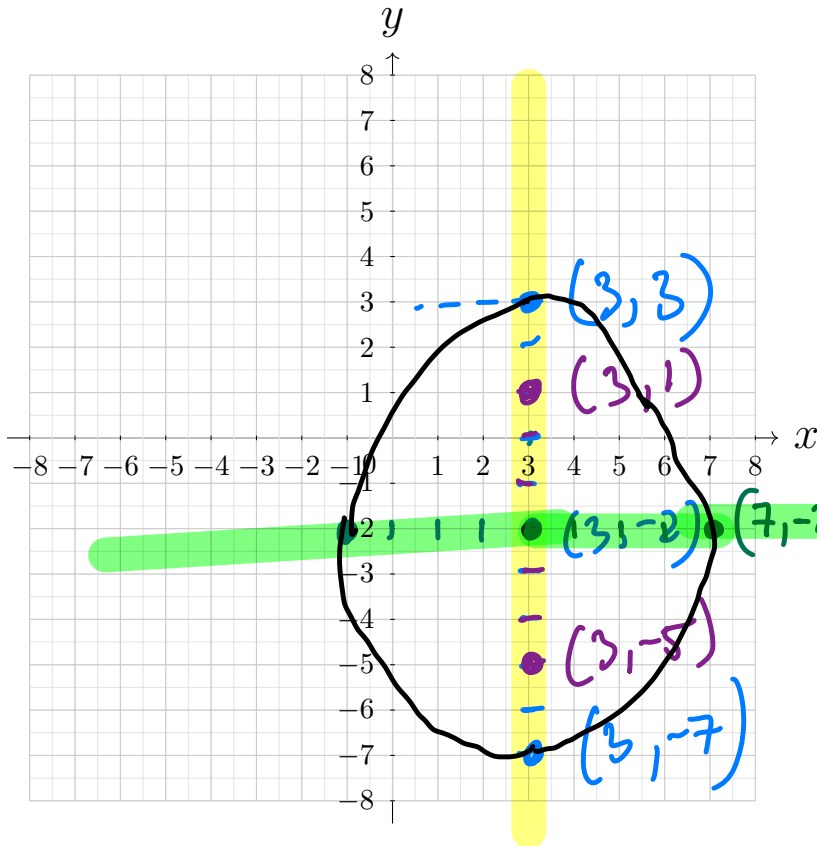
$$y^2 = 12 \left(1 - \frac{x^2}{16} \right)$$

$$2) \quad y = \pm \sqrt{12 \left(1 - \frac{x^2}{16} \right)}$$

4. Graphing and Ellipse centered at (h, k)

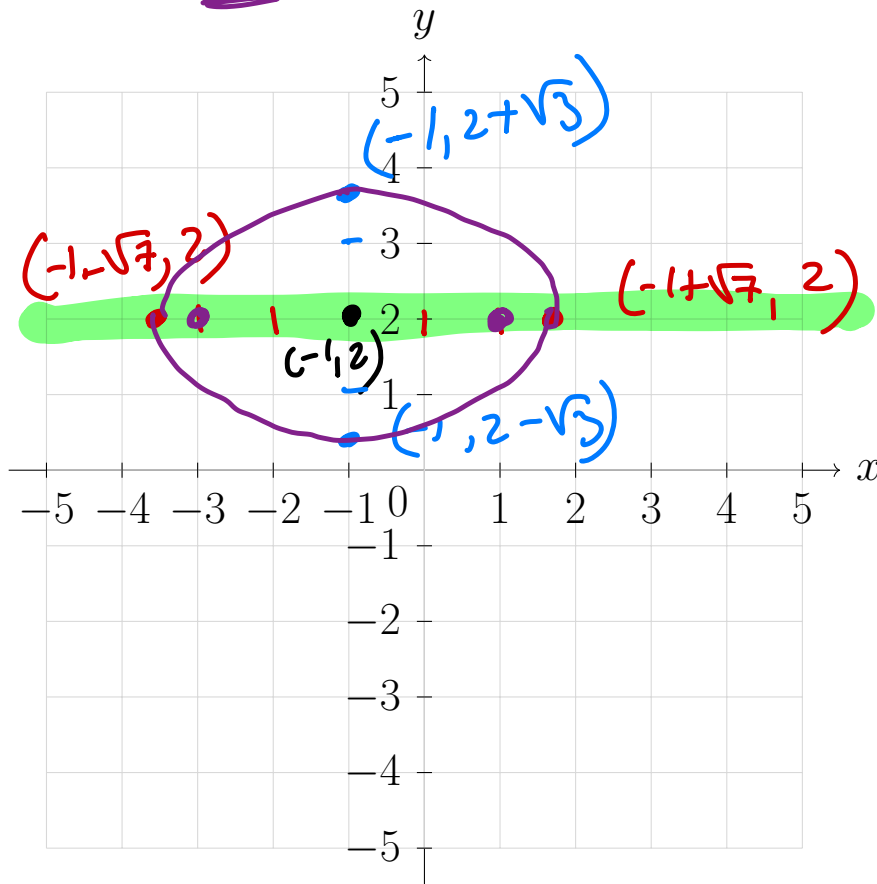
Replacing x by $x - h$ and y by $y - k$ shifts the graph of an ellipse h units horizontally and k units vertically.

Example 8: Given $\frac{(x - 3)^2}{16} + \frac{(y + 2)^2}{25} = 1$, graph the ellipse and identify the center, foci and vertices.



center: $(3, -2)$
elongated vertically
 $a = 5$
 $b = 4$
vertices: $(3, 3)$ & $(3, -7)$
foci: $(3, 1)$ & $(3, -5)$
need c
 $c^2 = a^2 - b^2$
 $= 25 - 16 = 9$
 $c = 3$

Example 9: Practice: Given $3x^2 + 7y^2 + 6x - 28y + 10 = 0$, write the equation of the ellipse in standard form, graph the ellipse and identify the center, foci and vertices.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{or } \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

• elongated horizontally
 • center $(-1, 2)$
 - foci $c^2 = a^2 - b^2$
 $= 7 - 3$
 $= 4$

$$3x^2 + 7y^2 + 6x - 28y + 10 = 0$$

$$3x^2 + 6x + 7y^2 - 28y = -10$$

$$3(x^2 + 2x + 1) + 7(y^2 - 4y + 4) = -10 + 3 + 28$$

$$3(x+1)^2 + 7(y-2)^2 = \frac{21}{21}$$

$$\frac{(x+1)^2}{7} + \frac{(y-2)^2}{3} = 1$$

$a = \sqrt{7}$ $b = \sqrt{3}$