Chapter 6 Matrices and Systems of Linear Equations

1. System of linear equations

In this section we will review systems of linear equations in two variables and then introduce matrices which are going to help up solve systems of linear equations with 3 and more variables.

Solve the system of linear equations using substitution or elimination:

1. $\begin{cases} 2x - y = 5 \\ x + 4y = 7 \end{cases}$ → line, → lines

solve for y
$-y = -2x + 5$
$y = 2x - 5$ ⇒ $2\cdot 3 - 5$ ⇒ $y = 1$

$x + 4(2x - 5) = 7$
$x + 8x - 20 = 7$ ⇒ $9x = 27$
$x = 3$

To solve a systems means to find (x, y) that work for all equations.
Is $(3, 1)$ a solution of this system?
x, y
$2\cdot 3 - 1 \stackrel{?}{=} 5$ yes $(3, 1)$ is a solution
$3 + 4\cdot 1 \stackrel{?}{=} 7$ yes

Substitution method: substitute one of the variables in terms of the other.

2. $\begin{cases} 2x + y = 1 \quad (-4) \\ 3x + 4y = 14 \end{cases}$

Elimination Method: eliminate one of the variable by multiplying one/both of the equation by a constant and then adding the equations.

$\begin{cases} -8x - 4y = -4 \\ 3x + 4y = 14 \end{cases}$

(+)

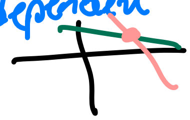
$-5x = 10$

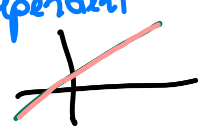
$x = -2$

Now plug $x = -2$ in first equation:
$2(-2) + y = 1$
$-4 + y = 1$ ⇒ $y = 5$

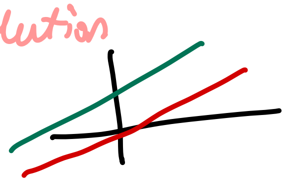
Solution is $(-2, 5)$.

Consistent system:
it has at least a solution
exactly one solution → independent
infinetly many solutions → dependent





Inconsistent System:
↳ no solution
parallel lines



2. Matrices

A matrix is simply a rectangular array of numbers. Matrices are used to organize information into categories that correspond to the rows and columns of the matrix.

We use capital letters to denote matrices: A, B, C, M, I...

$$A = \begin{array}{c} \begin{array}{ccc} \text{col 1} & \text{col 2} & \text{col 3} \end{array} \\ \begin{bmatrix} 2 & 5 & -3 \\ 1 & 0 & 10 \end{bmatrix} \end{array} \begin{array}{l} \to \text{row 1} \\ \to \text{row 2} \end{array} \qquad B = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 1 & 2 & 7 \\ 1 & 0 & 0 \\ 1 & 5 & -3 \end{bmatrix}$$

name

size of A is $\underbrace{2 \times 3}_{2 \text{ by } 3}$

row columns

size of B is 5×3

$b_{22} = -1$

$a_{21} = 1$

$a_{23} = 10$

$a_{12} = 5$

DEFINITION OF MATRIX

An $m \times n$ **matrix** is a rectangular array of numbers with m **rows** and n **columns**.

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}}_{n \text{ columns}} \Bigg\} m \text{ rows}$$

(size)

We say that the matrix has **dimension** $m \times n$. The numbers a_{ij} are the **entries** of the matrix. The subscript on the entry a_{ij} indicates that it is in the ith row and the jth column.

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{3 \times 3}$$

square matrix if $m = n$
rows = # cols

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O_2$$ zero 2x2 matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3} = I_3$$ 1s on main diagonal

called the identity matrix

3. Operations with Matrices

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We can add, subtract and multiply matrices(either multiply a matrix by scalar or multiply it by another matrix.

Consider the following matrices:
Carry out each indicated operation, or explain why it cannot be performed.

$$A = \begin{bmatrix} 2 & -3 \\ 0 & 5 \\ 7 & -\frac{1}{2} \end{bmatrix}_{3\times 2} \qquad B = \begin{bmatrix} 1 & 0 \\ -3 & 1 \\ 2 & 2 \end{bmatrix}_{3\times 2}$$

$$C = \begin{bmatrix} 7 & -3 & 0 \\ 0 & 1 & 5 \end{bmatrix} \qquad D = \begin{bmatrix} 6 & 0 & -6 \\ 8 & 1 & 9 \end{bmatrix}_{2\times 3}$$

2×3

Add/subtract matrices of same dimension.

$(a) A + B$ $= \begin{bmatrix} 2+1 & -3+0 \\ 0-3 & 5+1 \\ 7+2 & -\frac{1}{2}+2 \end{bmatrix}_{3\times 2} = \boxed{\begin{bmatrix} 3 & -3 \\ -3 & 6 \\ 9 & 3/2 \end{bmatrix}}_{3\times 2}$

$(b) C - D$ $= \begin{bmatrix} 7-6 & -3-0 & 0-(-6) \\ 0-8 & 1-1 & 5-9 \end{bmatrix}_{2\times 3} = \begin{bmatrix} 1 & -3 & 6 \\ -8 & 0 & -4 \end{bmatrix}$

$(c) C + A$ not possible to add C is 2×3 & A is 3×2.

$(d) 5A$ $= 5\begin{bmatrix} 2 & -3 \\ 0 & 5 \\ 7 & -1/2 \end{bmatrix} = \begin{bmatrix} 10 & -15 \\ 0 & 25 \\ 35 & -5/2 \end{bmatrix}$

Calculator:

$(e) AC$ $\quad A_{3\times 2} \quad C_{2\times 3}$ = answer is 3×3

$(f) AB$ $\quad A_{3\times 2} \quad B_{3\times 2}$ = not possible

$$A\,C = \begin{bmatrix} 2 & -3 \\ 0 & 5 \\ 7 & -1/2 \end{bmatrix}_{3\times \cancel{2}} \begin{bmatrix} 7 & -3 & 0 \\ 0 & 1 & 5 \end{bmatrix}_{\cancel{2}\times 3}$$

$$AC = \begin{bmatrix} 2\cdot 7 + (-3)0 & 2\cdot(-3) + (-3)(1) & 2\cdot 0 + (-3)5 \\ 0\cdot 7 + 5\cdot 0 & 0(-3) + 5(1) & 0\cdot 0 + 5\cdot 5 \\ 7\cdot 7 + (-1/2)\cdot 0 & 7(-3) + (-1/2)1 & 7\cdot 0 + -\frac{1}{2}\cdot 5 \end{bmatrix} = \begin{bmatrix} 14 & -9 & -15 \\ 0 & 5 & 25 \\ 49 & -\frac{43}{2} & -\frac{5}{2} \end{bmatrix}$$