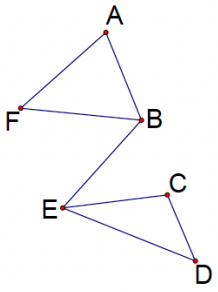
Section 1.2 Euler Paths and Circuits

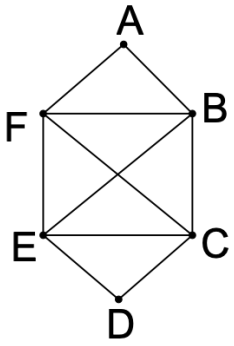
Euler Path

Example 1. Find an Euler path in the following graph.



Euler Circuit

Example 2. Find an Euler circuit in the following graph.



Euler's Circuit Theorem

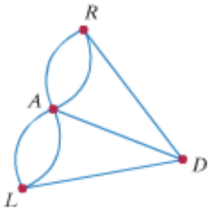
- If a graph is connected and every vertex has

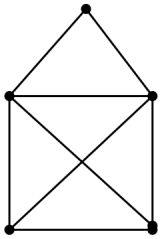
- If a graph has any vertices

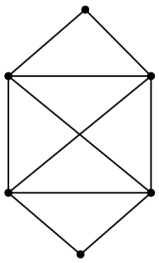
Euler's Path Theorem

- If a graph is connected and has exactly two vertices, then it has at least one Euler Path. The path must start at one of the vertices and end at the other.
- If a graph has more than two vertices, then it cannot have an Euler Path.

Example 3. Which of these graphs have an Euler's circuit or an Euler's Path?



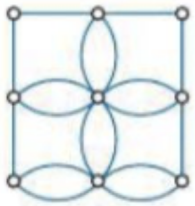




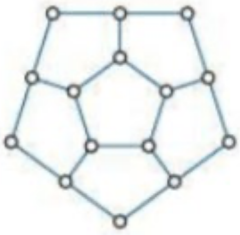
Example 4. For each of the three graphs below choose from one of the following answers and provide a short explanation for your answer.

1. The graph has an Euler circuit.
2. The graph has an Euler path.
3. The graph has neither an Euler circuit nor an Euler path.
4. The graph may or may not have an Euler circuit.
5. The graph may or may not have an Euler path.

(a)

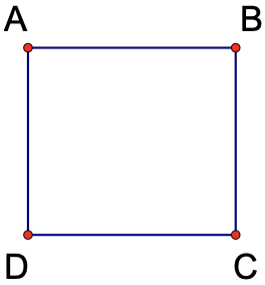


(b)



(c) A graph with eight vertices: six vertices of degree 2 and two vertices of degree 3.

Example 5. Is it possible to add an edge so that you get just one odd vertex?



Euler's Sum of Degrees Theorem

- The sum of the degrees of all the vertices of a graph equals

- A graph has always an

Now we know how to determine if a graph has an Euler circuit, but if it does, how do we find one? While it usually is possible to find an Euler circuit just by pulling out your pencil and trying to find one, the more formal method is Fleury's algorithm.

Fleury's algorithm for finding an Euler Circuit or Path

1. Determine the number of odd vertices in the graph. (If the graph is not connected, there is no Euler Circuit or Euler Path.)

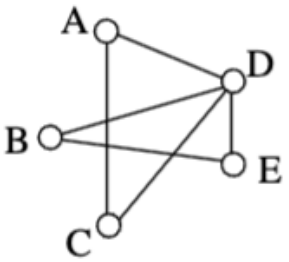
2. Select a starting vertex.

3. At each step, choose any edge leaving your current vertex but if you have a choice, don't choose a bridge of the yet-to-be-traveled part of the graph.

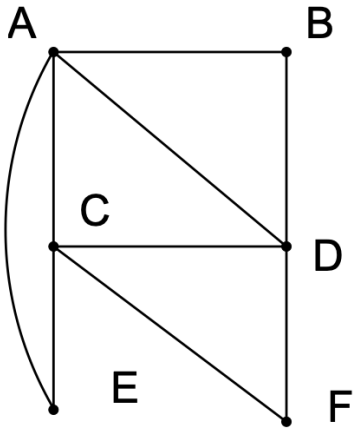
4. Add that edge to your circuit, and delete it (mark it) from the graph.

5. Continue until you're done. (covered all the edges).

Example 6. Find an Euler Circuit in the following graph using Fleury's Algorithm.



Example 7. Find an Euler Circuit in the following graph using Fleury's Algorithm.



Example 8. Find an Euler path in the following graph using Fleury's Algorithm.