Mth 146: Day 3
Section 7.2 Trig Integrals

1 Part 1: New Concepts & Worked Examples

We will continue to learn techniques to find integrals of trigonometric functions. The goal is to use trigonometric identities and rewrite the integral in such a way that we can use a u-substitution, where the u-sub is a trigonometric function.

Often, simplifying/rewriting using trig identities will help. Some useful identities:

1. Pythagorean:

2. Power Reducing:

3. Product to sum:
   $\sin(A)\cos(B) = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
   $\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
   $\cos(A)\cos(B) = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

Sine $\times$ cosine integrals:

- Odd power of sine or cosine $\implies$ save one factor, then convert the rest using the Pythagorean identities

- Only even powers $\implies$ use power reducing formulas to reduce powers until the integral can be computed

Tangent $\times$ secant integrals:

- Even secant $\implies$ $u = \tan(x)$, so save a $du = \sec^2(x)$ factor and use $\sec^2(x) = 1 + \tan^2(x)$
- Odd tangent $\implies$ $u = \sec(x)$, so save a $du = \sec(x)\tan(x)$ and use $\tan^2(x) = \sec^2(x) - 1$

Integrals involving powers of $\sin(x)$ and $\cos(x)$

Example 1. $\int \sin^2(x)\,dx$

Example 2. $\int \sin^3(x)\,dx$

Example 3. $\int \sin^5(x)\cos^2(x)\,dx$

Example 4. $\int \sin^4(x)\cos^2(x)\,dx$

Example 5. $\int \tan^6(x)\sec^4(x)\,dx$

Example 6. $\int \tan^3(x)\sec(x)\,dx$

Example 7. $\int \sec^3(x)\,dx$

Example 8. $\int \sin(4x)\cos(5x)\,dx$

2 Part 2: Your Turn — Practice

Try the following individually or in small groups. Final answers are given as footnotes at the bottom of the page — only the final answer, not the steps, so use them to check your work rather than replace it.

1. $\int \cos^3(x)\ dx$.[1]

2. $\int \sin^4(x)\ dx$.[2]

3. $\int \tan^4(x)\ dx$.[3]

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[1] Answer: $\sin(x) - \frac{\sin^3(x)}{3} + C$.

[2] Answer: $\frac{3x}{8} - \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32} + C$.

[3] Answer: $\frac{\tan^3(x)}{3} - \tan(x) + x + C$.

4. $\int \sec^4(x)\tan^2(x)\ dx$.[4]

5. $\int_0^{\pi/2} \sin^2(x)\cos^3(x)\ dx$.[5]

[4] Answer: $\frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5} + C$.

[5] Answer: $\frac{2}{15}$.

3 Part 3: Reflection

For each learning outcome below, rate how comfortable you feel right now, and write a few sentences: What clicked? What's still fuzzy? What would help?

1. **Use a combination of trigonometric identities, u-subs, and integration by parts to evaluate integrals involving products of powers of trigonometric functions.**

   Comfort level (circle one): 1 — not comfortable 2 3 4 5 — very comfortable

   Reflection:

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2. **Recognize which reduction strategy to use (splitting off one factor for a u-sub, vs. half-angle/double-angle reduction) based on whether the relevant power is odd or even.**

   Comfort level (circle one): 1 — not comfortable 2 3 4 5 — very comfortable

   Reflection:

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3. **Evaluate $\int \sec^3(x)\,dx$ and explain why it requires a different technique than the other secant/tangent integrals.**

   Comfort level (circle one): 1 — not comfortable 2 3 4 5 — very comfortable

   Reflection:
