

Math 140: Tuesday, Nov 12th, Day 20 Notes

Agenda for Tuesday, Nov 12th, 2024

Start Logic Chapter 5

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Coming up

- ! Online HW 12 due today
- ! Exam 3 next week on Thursday (material: starting at Day 15 - counting methods and including everything up until the exam)
- Deadline to withdraw from class is today

Statements

A statement is a declarative sentence that is either true or false but not both true and false at the same time. Each statement has a truth value. It is either True or False.

Examples of statements:

1. Barack Obama was the president of the USA. T
2. The number 2 is even and less than 10. T
3. Jimmy Fallon was responsible for the development of graph theory. F

Examples that are not statements:

1. What is your name? No. It's a question
2. Pick that up! a command
3. This statement is false. This is an example of a paradox.

A statement that or situation that contradict itself or defies intuition.

Liar's paradox.
If true, then it's false.
If it's false, then it's true.

Compound Statements

A compound statement is a statement that combines statements using logical connectors such as:

not, and, or, if ... then, if and only if.

Examples of compound statements:

1. If you eat your vegetables, then you get a cookie.
2. Tom is taking MTH 140 or MTH 141.
3. I biked to work and I carried my lunch.

Not a compound statement: Phineas and Ferb TV show was funny.

Symbolic Logic

The goal of S.L. is to translate statements into symbols, analyze or simplify, then translate back to words

Use lower case letters for simple statements.

Example:

p:

r:

Truth Tables

A truth table is a table that shows all possibilities for the truth values of a statement.

P
T
F

Negation - denote it $\sim$

The negation of a true statement is false and the negation of a false statement is true. (The statement and its negation make up all the possibilities).

Examples – Write the negation of each statement:

- p: My car is a Ford.

$\sim p$: is not

- q: Today is not Thursday.

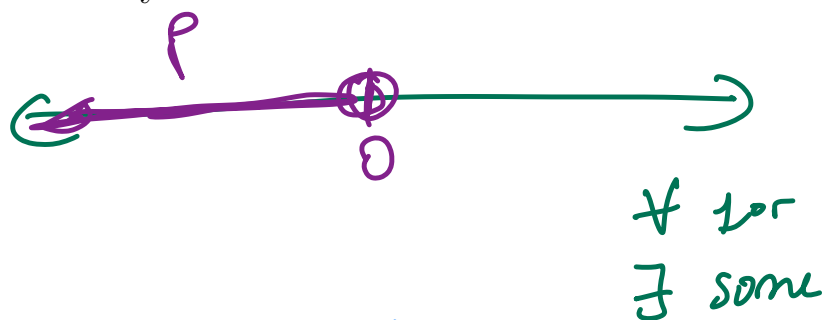
$\sim q$: is

- r: $6x < 0$

$6x \geq 0$

- s: $19y \leq -3$

$\sim s$: $y >$



Quantifiers

Words such as all, none, some, _____ are called quantifiers.

Note: The word "some" in this context means least one.

Examples of statements with quantifiers - Write the negation of each statement:

1. All cars have airbags.

Some cars don't have an airbag
if at least one car

2. Some cars have antilock brakes.

No cars have

3. No cars have window stickers.

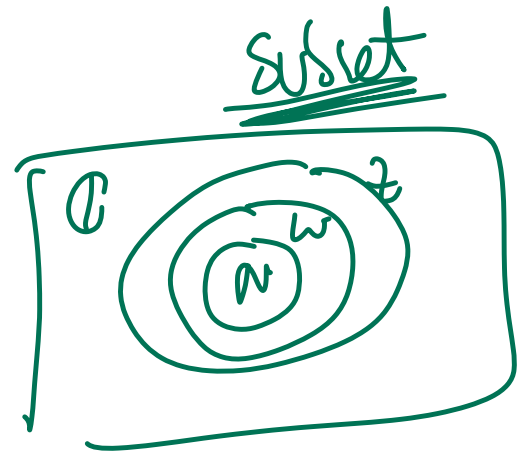
At least one car has w-s-

4. Some cars do not have air conditioning.

All cars have AC.

5. At least one car has a bumper sticker.

No car has.



Summary

p

$\sim p$

$\forall$ all are

$\exists$ some are
some are not
none are

$\exists$ not some (at least one) or not
 $\forall$ not none are
all are
some are

Sets of numbers:

Natural num (or count): $N = \{1, 2, 3, 4, \dots\}$

whole num $\{0, 1, 2, \dots\}$

Inte $\mathbb{Z}$

Rat $= \left\{ \frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0 \right\}$

Real $= \{x \mid x \text{ is a number that can be written as decimal.}\}$

Irr $= \{x \mid x \text{ is a real \# and } x \text{ can't be written as a quotient of integers}\}$

True or False. If False, give a counter example.

A counterexample is an example that shows that the statement does not hold.

1. Every natural number is an integer. T
2. There exists an integer that is not a natural number. T
3. All irrational numbers are real numbers. T
4. Some whole numbers are not rational numbers. F
5. Each rational number is a positive number. F -

Conjunction and Disjunction

Conjunction: "and"

Symbol:

Disjunction: (inclusive "or" meaning A or B or both)

Symbol:

Example: Let p : The dog is large. q : The dog is fluffy.

Translate from symbols to words:

1. $\neg q$

2. $\neg(q)$

3. $p \vee q$

4. $p \wedge q$

5. $\neg(p \vee q)$

6. $\neg(p \wedge q)$

Translate from words to symbols:

1. The dog is large or the dog is not fluffy.
2. The dog is neither large nor fluffy.