Math 140: Thursday Aug 22nd, Day 2 Notes

Agenda for Thursday, Aug 22nd,2024	
Continue Intro to Graph Theory notes	**1**
Section 1.2 Euler Paths and Circuits	**1**

Reminders

- ! Turn in Group activity 0
- First Canvas Homework will open today and be due by next Thursday
- Office Hour **today 1-2:15pm** in C162
- Culinary event - mini tostadas in B019 from 3-5pm today

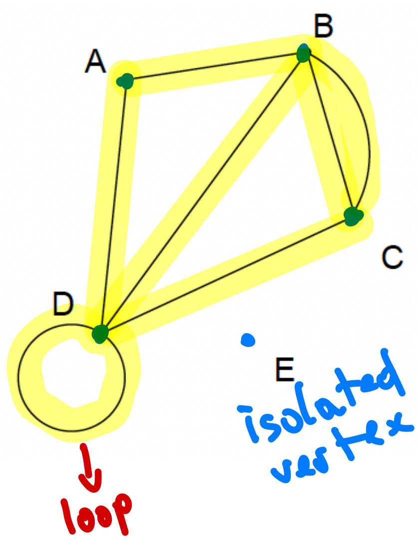
Intro to graph theory: continued

Recall: Graph, vertices, edges

Intro to graph Theory and Euler Circuits and Paths

Graph - A picture that contains a set of vertices and a set of edges.

Example 1. Consider the graph below. Give the set of vertices and the set of edges:

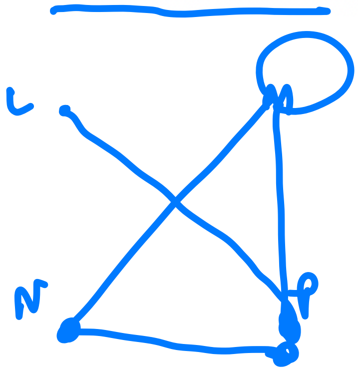


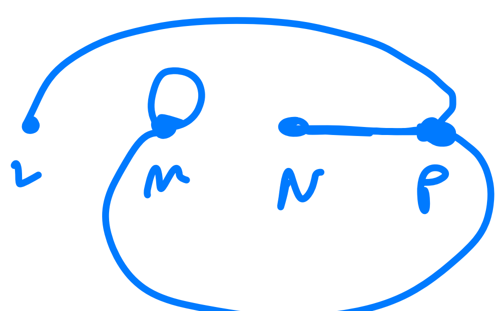
$V = \{A, C, B, D, E\}$

$E = \{AB, BC, AD, DC, BC, DD\}$

Day 2 :

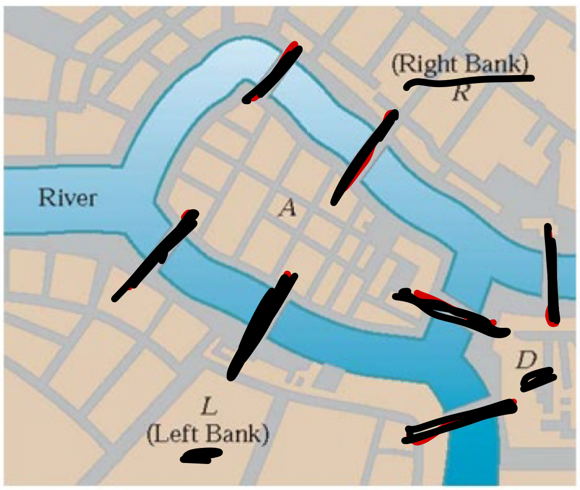
Example 2. Draw two different graphs for
$\mathcal{V} = \{L, M, N, P\}$ and $\mathcal{E} = \{LP, MM, PN, MN, PM\}$



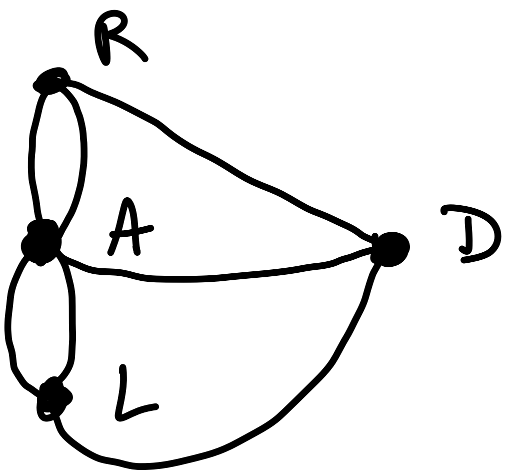


These two graph are the same (equivalent)
- same vertices
- same edges

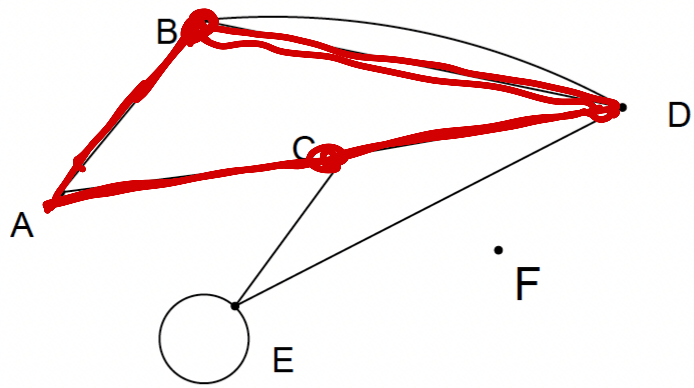
Example 3. Back in the 18th century in the Prussian city of Königsberg, a river ran through the city and seven bridges crossed the forks of the river. The river and the bridges are highlighted in the picture to the right. As a weekend amusement, townsfolk would see if they could find a route that would take them across every bridge once and return them to where they started.



Draw the graph modeling this problem.



Example 4. For next terminology consider the following graph:



Adjacent vertices - vertices connected by an edge.

ex: A and B are adjacent.
B and E are not. Also A and D are not.

Adjacent edges - edges that share a vertex.

ex: BA and AC are adjacent edge.
ED and BD not adj. edje.

Degree of a vertex – the number of edges at that vertex. A loop contributes twice to a degree!

deg(A) = 2 deg(C) = 3 deg(E) = 4
deg(B) = 3 deg(D) = 4 deg(F) = 0

Path – a sequence of vertices that describe a trip along the edges of a graph so that you never travel an edge more than once. The **length** of a path is the number of edges in the path.

ex: ABD is a path of length 2
CEDB is another path of length 3

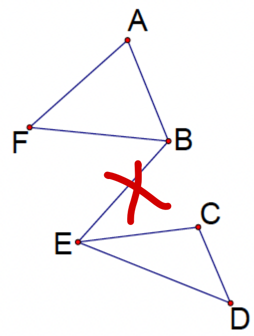
Circuit – a path that starts and ends at the same vertex.

ex: BACDB

Connected graph – a connected graph contains a path between any two vertices.

Ex 4 graph not connected.
because of F.

Example 5. Consider the following graph.



EB is a bridge.

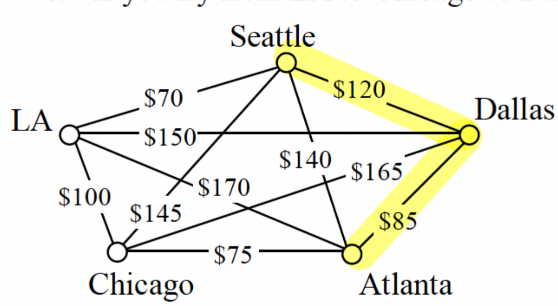
A bridge is an edge that if it were erased from a connected graph it would make the graph disconnected.

Weighted Graphs

Depending upon the problem being solved, sometimes weights are assigned to the edges. The weights could represent the distance between two locations, the travel time, or the travel cost. It is important to note that the distance between vertices in a graph does not necessarily correspond to the weight of an edge.

Try it Now 1

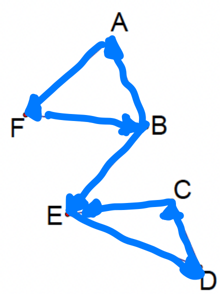
1. The graph below shows 5 cities. The weights on the edges represent the airfare for a one-way flight between the cities.
   a. How many vertices and edges does the graph have? 5 vertices 10 edges
   b. Is the graph connected? yes
   c. What is the degree of the vertex representing LA? 4
   d. If you fly from Seattle to Dallas to Atlanta, is that a path or a circuit?
   e. If you fly from LA to Chicago to Dallas to LA, is that a path or a circuit?



Section 1.2 Euler Paths and Circuits

Euler Path -is a path that passes through every edge of a graph exactly once.

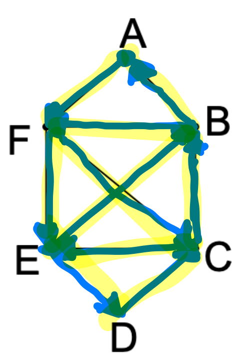
Example 1. Find an Euler path in the following graph.



BAFBEDCE

Euler Circuit is a circuit that passes through every edge of a graph exactly once.

Example 2. Find an Euler circuit in the following graph.



AFEDCEBFCBA

starts at A and ends at A
covers each edge only once.

Euler's Circuit Theorem

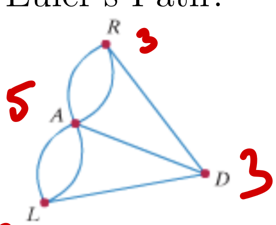
- If a graph is connected and every vertex has even degree, then it has at least one Euler circuit.

- If a graph has any vertices with odd degree, it does not have an Euler circuit.

Euler's Path Theorem

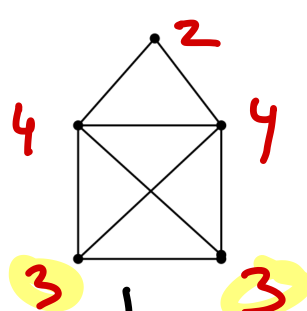
- If a graph is connected and has exactly two odd degree vertices, then it has at least one Euler Path. The path must start at one of the odd degree vertices and end at the other.
- If a graph has more than two odd degree vertices, then it cannot have an Euler Path.

Example 3. Which of these graphs have an Euler's circuit or an Euler's Path?

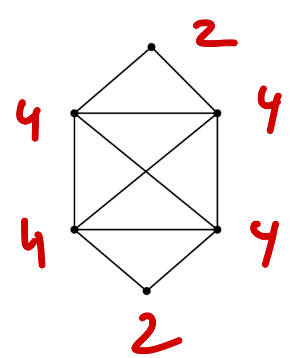


none

since the degrees of all the vertices are odd



has Euler path
since we have only two vertices with odd degree



this is example 2 from previous page.