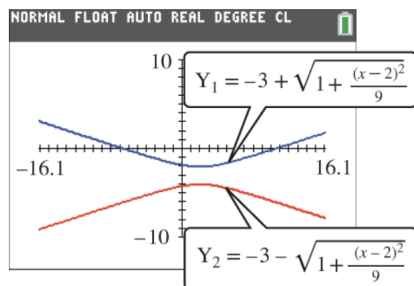
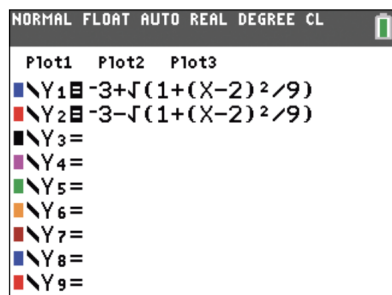


Calculator Instructions:



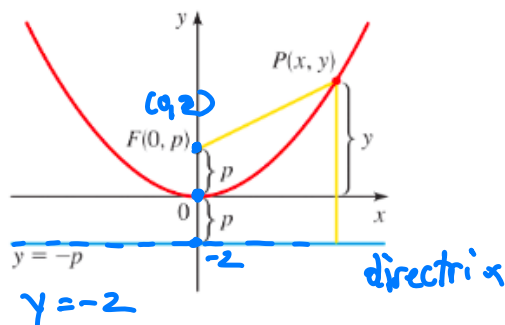
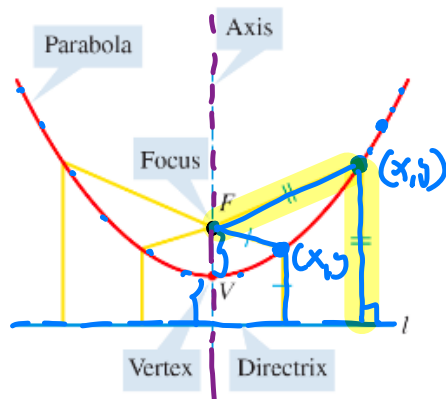
$$y = ax^2 + bx + c$$

5.9.1 Continued: Parabolas

In this section we study **parabolas** from a **geometric** rather than an **algebraic point of view**. We will start with parabolas that are situated with the **vertex** at the origin and that have a **vertical** or **horizontal axis of symmetry**.

GEOMETRIC DEFINITION OF A PARABOLA

A **parabola** is the set of all points in the plane that are **equidistant** from a fixed point F (called the **focus**) and a fixed line l (called the **directrix**).



PARABOLA WITH VERTICAL AXIS

The graph of the equation

$$x^2 = 4py$$

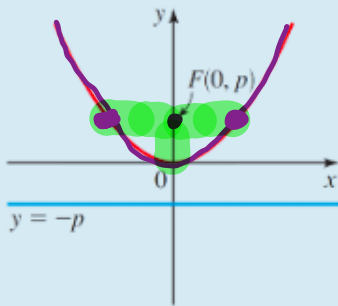
is a parabola with the following properties.

VERTEX $V(0, 0)$

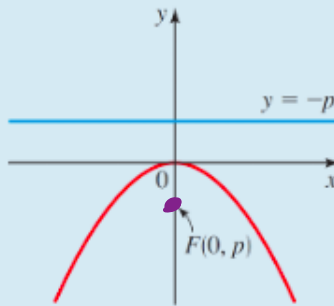
FOCUS $F(0, p)$

DIRECTRIX $y = -p$

The parabola opens upward if $p > 0$ or downward if $p < 0$.



$x^2 = 4py$ with $p > 0$



$x^2 = 4py$ with $p < 0$

PARABOLA WITH HORIZONTAL AXIS

The graph of the equation

$$y^2 = 4px$$

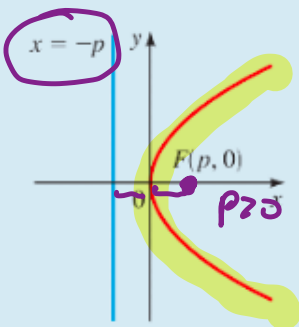
is a parabola with the following properties.

VERTEX $V(0, 0)$

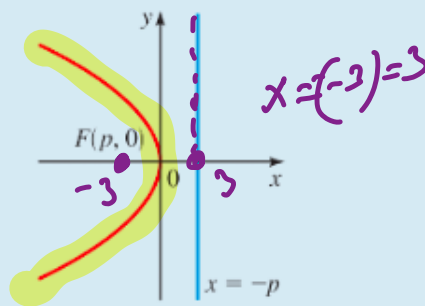
FOCUS $F(p, 0)$

DIRECTRIX $x = -p$

The parabola opens to the right if $p > 0$ or to the left if $p < 0$.



$y^2 = 4px$ with $p > 0$

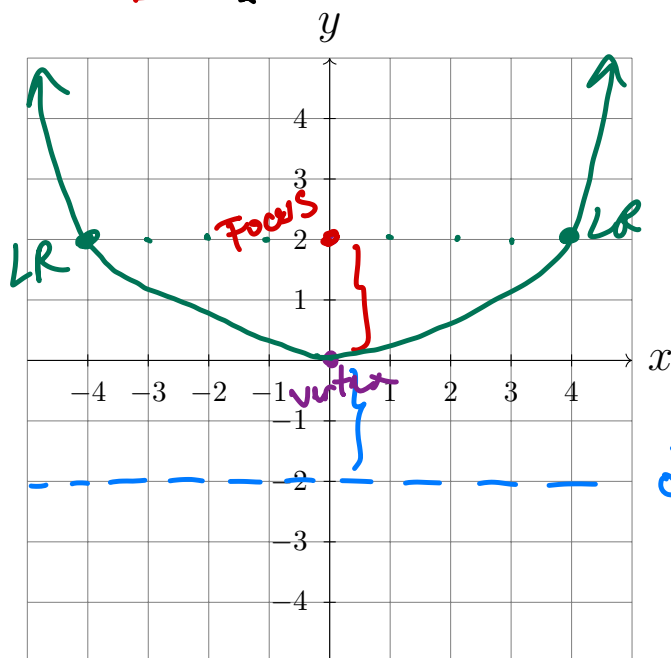


$y^2 = 4px$ with $p < 0$

Latus rectum endpoints: are the points on the parabola that $2|p|$ units from the focus.

Note: The parabola always opens towards the focus and away from the directrix!!!

Example 1: Find an equation for the parabola with vertex $V(0, 0)$ and focus $F(0, 2)$, and sketch its graph.



equation:

$$x^2 = 4py$$

$p = 2 > 0$ opens up

Latus rectum:

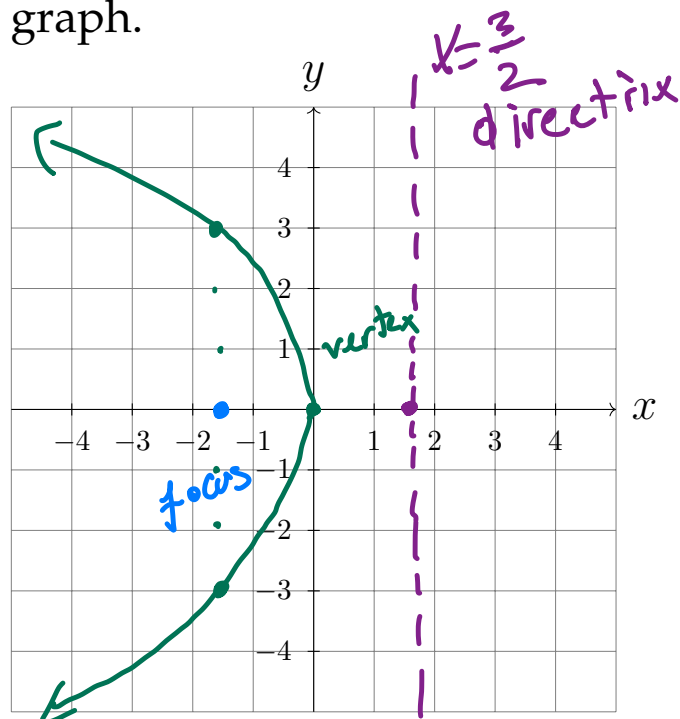
$$2|p| = 2(2) = 4 \text{ units from the focus.}$$

directrix
 $y = -2$

Ans: $x^2 = 4 \cdot 2 y \Rightarrow x^2 = 8y$
 $y = \frac{1}{8}x^2$

Example 2: A parabola has the equation $6x + y^2 = 0$.

Find the focus, focal diameter and directrix of the parabola and draw the graph.



$$6x + y^2 = 0$$

$$y^2 = -6x$$

$$y^2 = 4px$$

Solve: $4p = -6$
 $p = -\frac{6}{4} \Rightarrow p = -\frac{3}{2}$

parabola opens to the left.

need LR $2|p| = 2|\frac{-3}{2}| =$
 $= 2 \cdot \frac{3}{2} = 3$

focal diameter: $4|p| = 4(\frac{3}{2}) = 6$

6. Graphing hyperbolas with different ~~center~~

vertex

The following summarizes the equation and features of hyperbolas centered at (h, k) .

$$x^2 = 4py$$

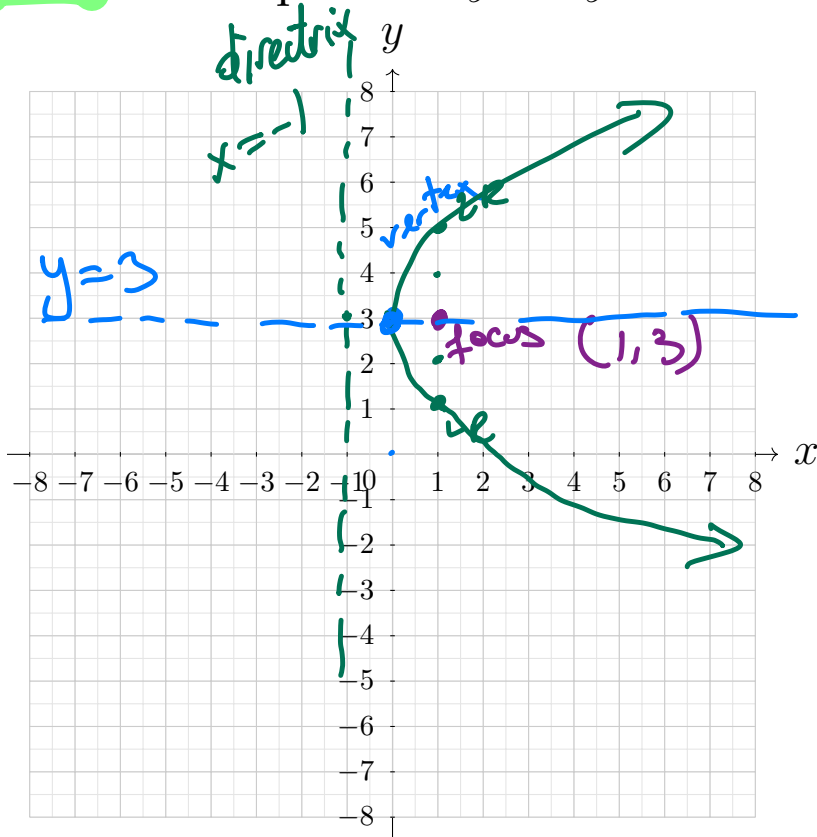
(h, k)

$$(x-h)^2 = 4p(y-k) \quad *$$

$$y^2 = 4px$$

$$(y-k)^2 = 4p(x-h) \quad *$$

Example 3: Identify the vertex, focus, directrix, axis of symmetry, latus rectum of the parabola $y^2 - 6y - 4x + 9 = 0$. Draw the parabola.



$$\begin{aligned}
 y^2 - 6y - 4x + 9 &= 0 \\
 y^2 - 6y &= 4x - 9 \\
 y^2 - 6y + 9 &= 4x - 9 + 9 \\
 (y-3)^2 &= 4(x-0) \\
 (h, k) &= (0, 3) \\
 &\text{vertex} \\
 p &= 1 > 0 \text{ opens to the right.} \\
 LR : 2|p| &= 2|1| = 2
 \end{aligned}$$