**Day 16: MTH 122**
*3.1 Quadratic Functionn and Applications*

Recall: Linear:
→ $ax+by=c$
→ $y=mx+b$

# 1 Quadratic Functions and Applications

## 1.1 Definition and Forms of Quadratic Functions

**Definition: Quadratic Function**

A **quadratic function** is a function of the form:

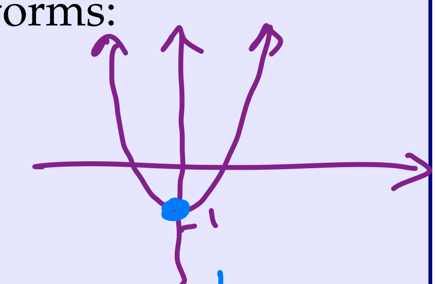
$$\boxed{f(x) = ax^2 + bx + c}$$

where $a$, $b$, and $c$ are real numbers, and $a \neq 0$.
The graph of a quadratic function is called a **parabola**.

**Definition: Forms of Quadratic Functions**

A quadratic function can be written in two main forms:
**General Form:**

$$f(x) = ax^2 + bx + c$$



**Vertex Form:**

$$f(x) = a(x-h)^2 + k$$

vertex

$(h,k)=(0,-1)$

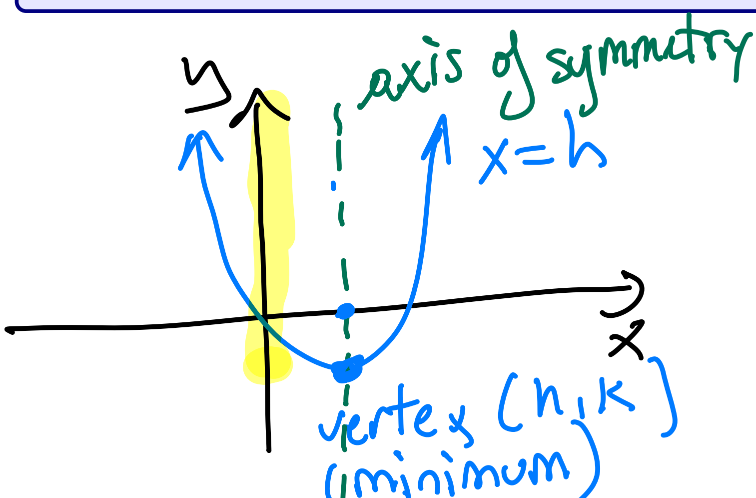
where $(h, k)$ is the vertex of the parabola.

x coord → y coordinate

## Definition: Properties of Quadratic Functions

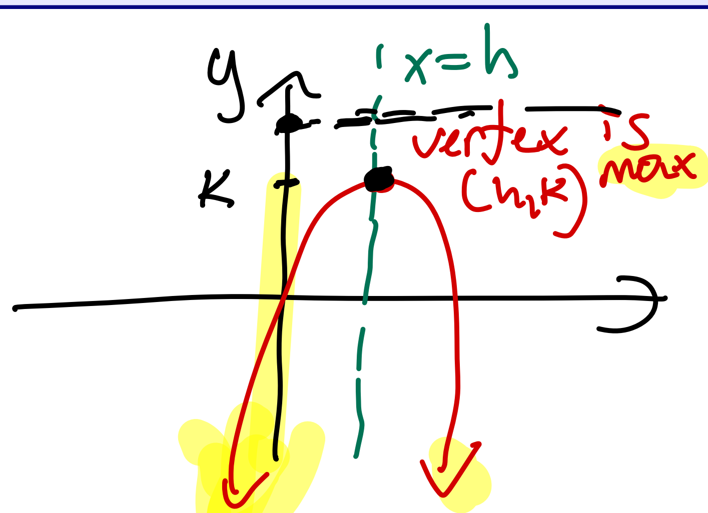
For a quadratic function $f(x) = a(x-h)^2 + k$ in vertex form:

1. **Opens upward** if $a > 0$, **opens downward** if $a < 0$.
2. The **vertex** is at the point $(h, k)$.
3. The graph has a **minimum** value of $k$ if $a > 0$.
4. The graph has a **maximum** value of $k$ if $a < 0$.
5. The **axis of symmetry** is the vertical line $x = h$.
6. The **domain** is all real numbers (i.e., $(-\infty, \infty)$).
7. The **range** is $[k, \infty)$ if $a > 0$, or $(-\infty, k]$ if $a < 0$.




a>0
opens up

domain: $(-\infty, \infty)$
range: $[k, \infty)$




a<0
opens down

domain: $(-\infty, \infty)$
range: $(-\infty, k]$

$f(x) = a(x-h)^2 + k$ vertex $(h,k)$

## Example 1: Analyzing a Quadratic Function in Vertex Form

Analyze the quadratic function $f(x) = -2(x-1)^2 + 8$ by finding:

a) Whether the parabola opens upward or downward

b) The vertex

c) The axis of symmetry

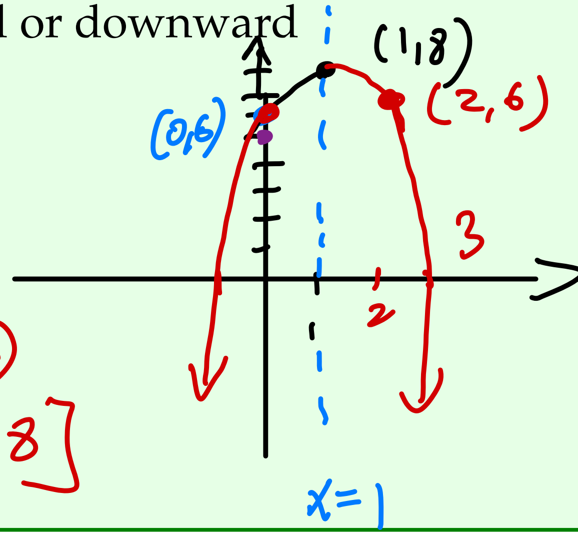
d) The maximum or minimum value

e) The $y$-intercept

f) The $x$-intercepts

g) The domain and range → $(-\infty, \infty)$ → $(-\infty, 8]$

h) A sketch of the graph




**Solution:**

a) down $a=-2<0$

b) vertex $(h,k) = (1,8)$

c) $x=1$ $(x=h)$

d) max value is 8 and it happens when $x=1$

e) y intercept : set $x=0$

$f(0) = -2(0-1)^2+8 = -2(1)+8 = 6$

f) $f(x) = -2(x-1)^2+8$

$0 = -2(x-1)^2+8$

$-2(x-1)^2+8=0$

$\frac{-2(x-1)^2}{-2} = \frac{-8}{-2}$

$(x-1)^2 = 4$

$x-1 = \pm\sqrt{4}$

$x-1 = \pm 2$ → $x = 1 \pm 2$

$x=3$

$x=-1$

## Example 2: Converting from General Form to Vertex Form

For the quadratic function $f(x) = 3x^2 + 12x + 5$:

$a=3$
$b=12$
$c=5$

a) Convert to vertex form by completing the square

b) Identify the vertex

$a=3>0$

c) Determine whether the parabola opens upward or downward

d) Find the axis of symmetry $x=-2$

e) Find the maximum or minimum value $(-2,-7)$

f) Find the $y$-intercept $(0,5)$

g) Find the $x$-intercepts (if any) $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

h) Determine the domain and range

i) Sketch the graph

**Solution:**

$f(x) = a(x-h)^2 + k$

$f(x) = 3x^2 + 12x + 5$

To bring it to vertex form we complete the square in x.

$f(x) = 3x^2 + 12x + 5$

$f(x) = 3(x^2 + 4x \quad) + 5$

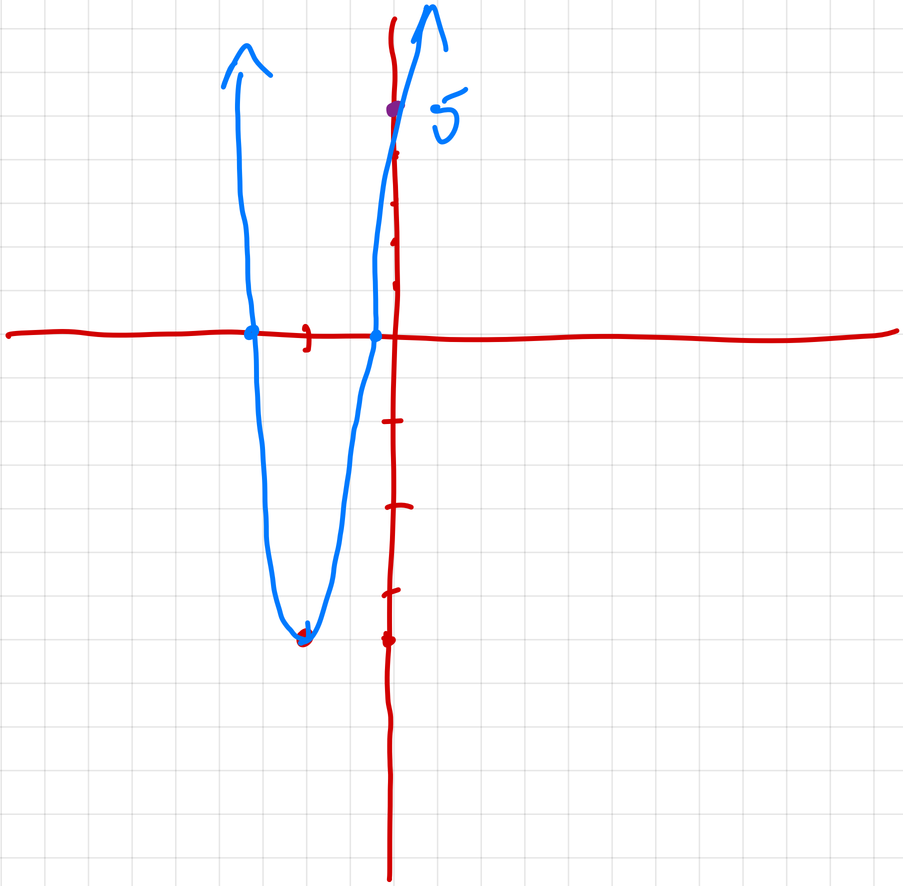
$12 + f(x) = 3(x^2 + 4x + 4) + 5$

$12 + f(x) = 3(x+2)^2 + 5$

$-12 \qquad -12$

$\boxed{f(x) = 3(x+2)^2 - 7}$ vertex form

$(h, k) = (-2, -7)$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### 1.3 The Discriminant and Number of Solutions

**Definition: The Discriminant**

For a quadratic equation $ax^2 + bx + c = 0$, the **discriminant** is defined as:

$$\Delta = b^2 - 4ac$$

The discriminant determines the number of real solutions to the equation:

- If $\Delta > 0$, then the equation has
- If $\Delta = 0$, then the equation has
- If $\Delta < 0$, then the equation has

**Note: Graphical Interpretation of the Discriminant**

Graphically, for a parabola $y = ax^2 + bx + c$:

- If $\Delta > 0$, the parabola intersects the $x$-axis at two distinct points.
- If $\Delta = 0$, the parabola is tangent to the $x$-axis (touches it at exactly one point).
- If $\Delta < 0$, the parabola doesn't intersect the $x$-axis (it lies entirely above or below the $x$-axis).

## Example 6: Projectile Motion

A ball is thrown upward from a height of 6 feet with an initial velocity of 64 feet per second. The height $h(t)$ of the ball (in feet) after $t$ seconds is given by:

$$\boxed{h(t) = -16t^2 + 64t + 6}$$

a) Find the maximum height reached by the ball. 9

b) At what time does the ball reach its maximum height? 2

c) When does the ball hit the ground? 4.092

d) Sketch the graph of the height function.

$a = -16 < 0$ opens down

y intercept: 6

x intercepts:

$a = -16, b = 64, c = 6$

$$x = \frac{\left(-64 \pm \sqrt{(64)^2 - 4(-16)(6)}\right)}{\left(2(-16)\right)} = \begin{cases} -0.092 \\ 4.092 \end{cases}$$