**Day 2 Notes**
**Wednesday February 5th**
*1.3 Graphs of Functions*

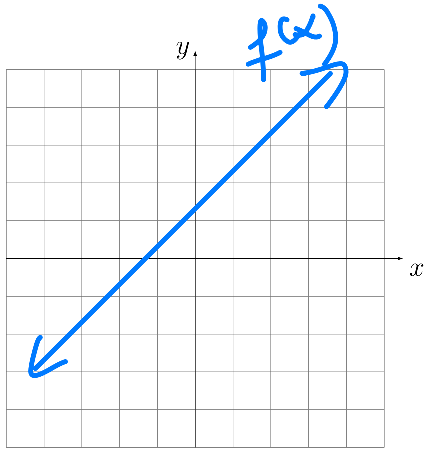
## 1. Graphs of Functions: Introduction

The most important way to visualize a function is through its graph. In this section we investigate in more detail the concept of graphing functions.

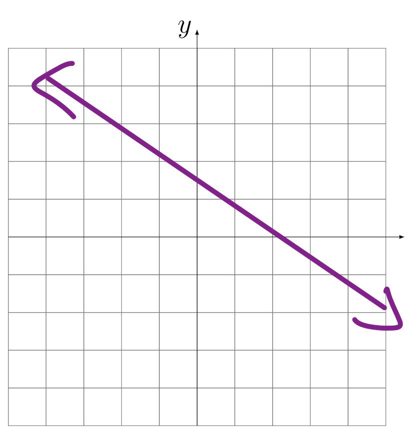
To graph of a function $f$, we plot the points $(x, y)$ in a coordinate plane where the $x$ coordinate represents an input and the $y$ coordinate is the corresponding output of the function, $y = f(x)$.

Examples of some functions and their graphs.

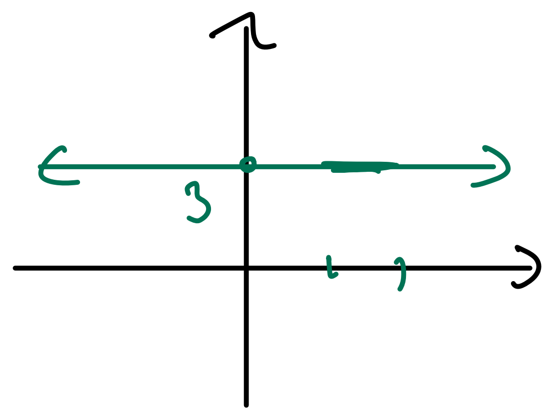
Linear Functions $f(x) = mx + b$; $m$ = slope, $b$ = y intercept



$m > 0$



$m < 0$

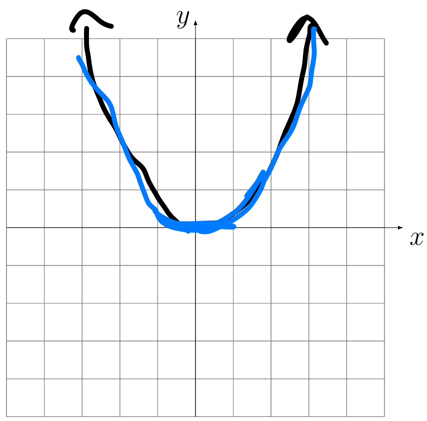


$m = 0$

## Power Functions - Positive Exponents

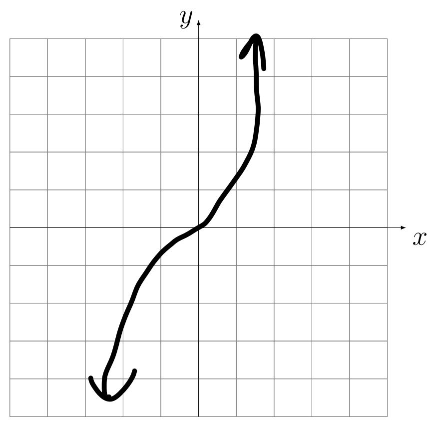
$f(x) = Kx^P$

$P$ - positive integer




$f(x) = x^2$

$f(x) = x^4$

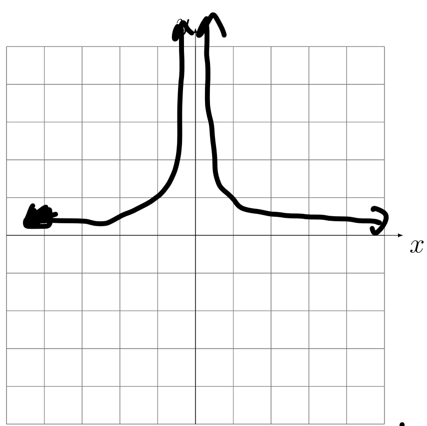



$f(x) = x^3$

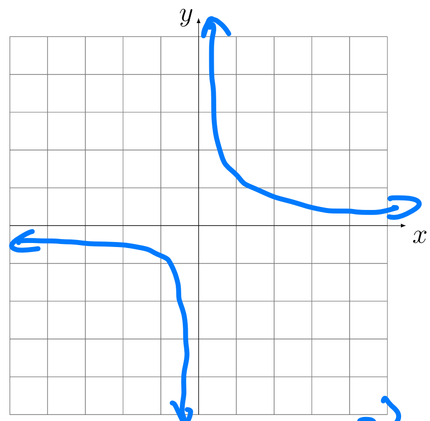
## Power Functions - Negative Exponents

$f(x) = Kx^P$

$P$ - negative integer

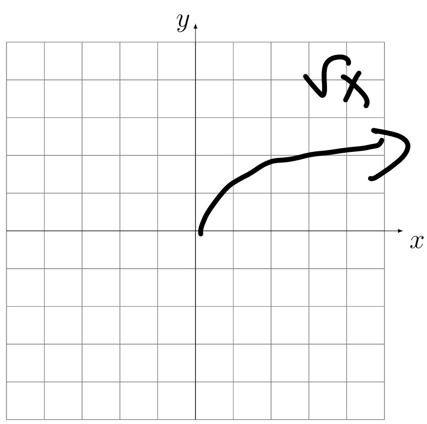



$f(x) = x^{-2} = \frac{1}{x^2}$

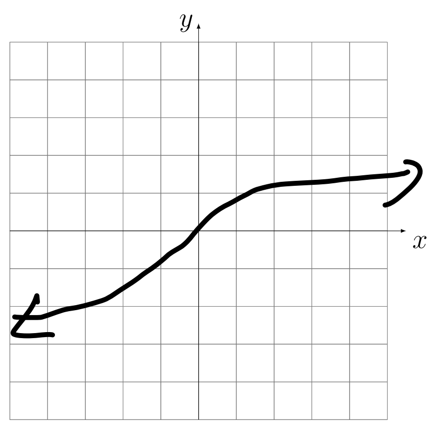



$f(x) = x^{-3} = \frac{1}{x^3}$

## Root Functions




$f(x) = \sqrt[2]{x}$




$f(x) = \sqrt[3]{x}$

## 2. Graphing Piecewise defined functions

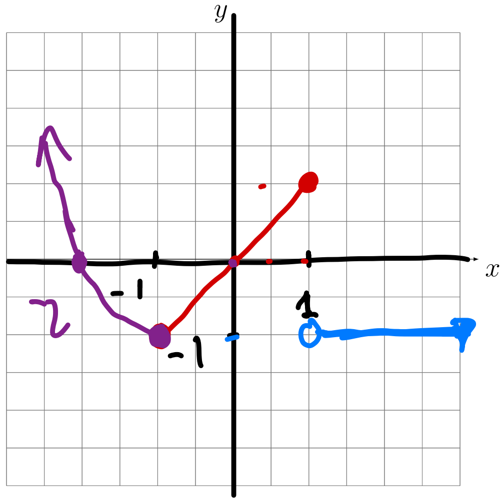
Graph the function

$$f(x) = \begin{cases} x^2 + 2x & x \le -1 \\ x & -1 < x \le 1 \\ -1 & 1 < x \end{cases}$$

parabola

line

horizontal line




parabola $1x^2+2x$

$x^2+2x = x(x+2)$

vertex: $\left(-\frac{b}{2a}, y\right)$

up $\vee$

$x^2+2x$

$ax^2+bx+c$

$a=1$

$b=2$

vertex: $-\frac{b}{2a} = -\frac{2}{2(1)}$

x coord $=-1$

y coord $= (-1)^2+(2)(-1)$

$= 1-2$

$=-1$

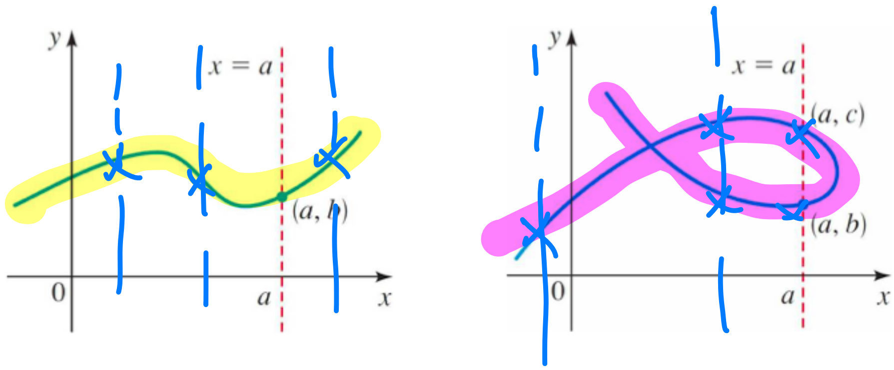
vertex $(-1,-1)$

Steps to graph a piecewise function:

1. Graph each piece on its given domain
2. Use solid dots (•) for included endpoints
3. Use open dots (○) for excluded endpoints
4. Check that the pieces connect as specified

## 3. Function or not a function?

Given a graph, one can decide if the graph represents a function if the graph passes the **vertical line test**: A curve in the xy-plane represents a function if and only if no vertical intersects the curve more than once.




Graph of a function    Not a graph of a function

Vertical Line Test

Given the equation of the relationship between $x$ and $y$, one can decide if $y$ represents a function of $x$ by solving for $y$ and seeing if each $x$ value has exactly one $y$ value assigned to it.

**Examples:** Does the equation define $y$ as a function of $x$?

1. $y - x^2 = 2$

$y = 2 + x^2$ yes

2. $x^2 + y^2 = 4$

$y^2 = 4 - x^2$ (square root both sides)

$\sqrt{y^2} = \pm\sqrt{4-x^2}$

$y = \pm\sqrt{4-x^2}$

$x = 0$

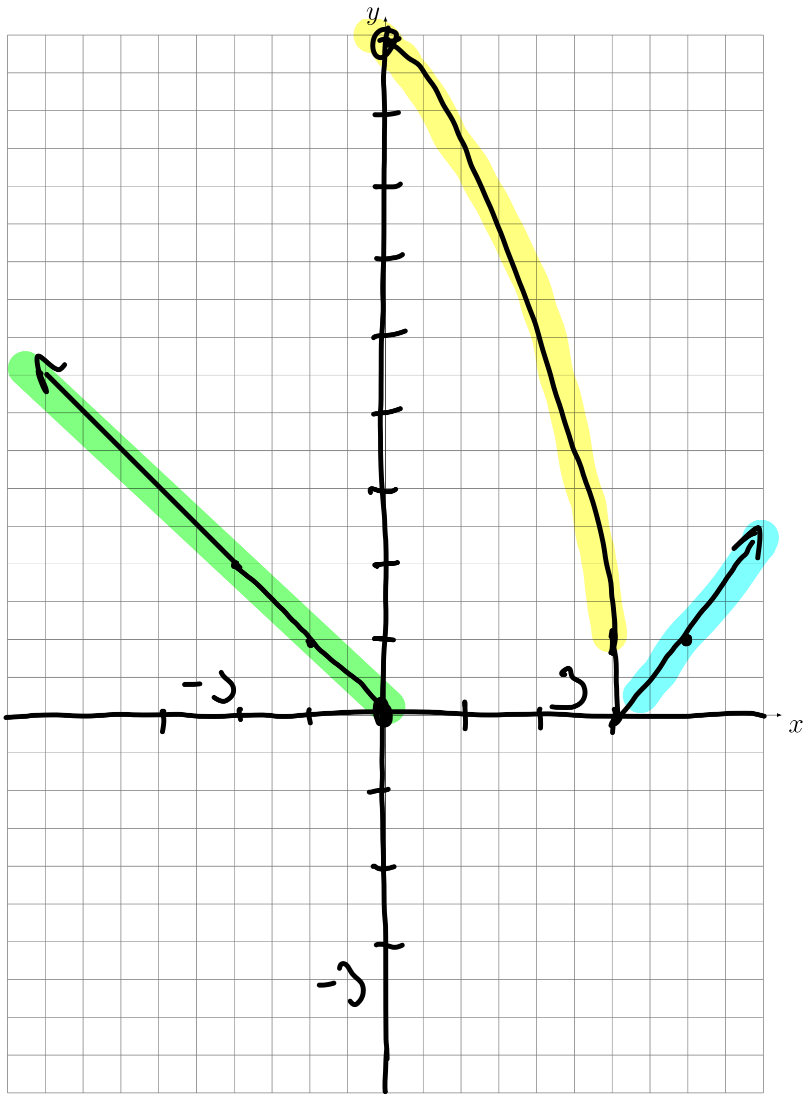
$y = \pm\sqrt{4-0^2} \Rightarrow y = \pm 2$

No!

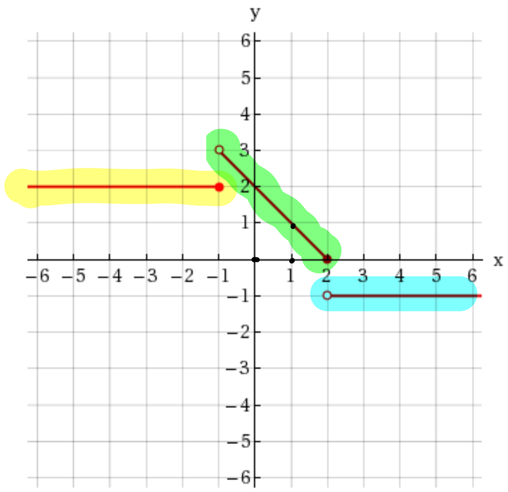
**Practice:** Graph the following function by plotting points.

$$f(x) = \begin{cases} -x & x \le 0 \\ 9 - x^2 & 0 < x \le 3 \\ x - 3 & 3 < x \end{cases}$$

line : m = -1, b = 0
parabola down
line : m = 1, b = -3



**Practice:** Find a formula for the function and state its domain and range.



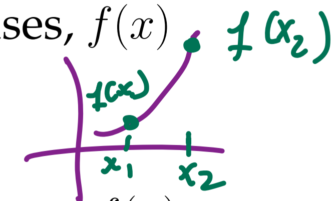

$$f(x) = \begin{cases} 2, & x \le -1 \\ -x+2, & -1 < x \le 2 \\ -1, & x > 2 \end{cases}$$

## Increasing/decreasing/constant function

Let $f$ be a function.

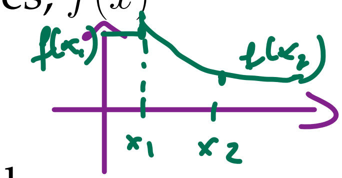
- A function is **increasing** on an interval if as $x$ increases, $f(x)$ increases.

  if $x_1 < x_2$ then $f(x_1) < f(x_2)$



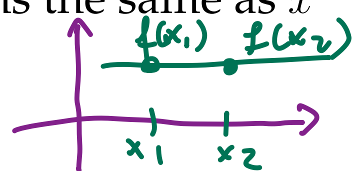

- A function is **decreasing** on an interval if as $x$ increases, $f(x)$ decreases.

  if $x_1 < x_2$ then $f(x_1) > f(x_2)$

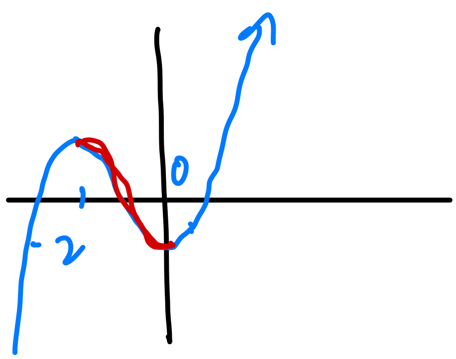



- A function is **constant** on an interval if $f(x)$ remains the same as $x$ changes.

  if $x_1 < x_2$ then $f(x_1) = f(x_2)$



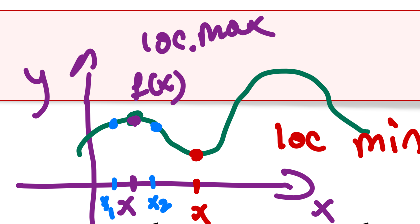

**Example 1.** Graph the function $f(x) = x^3 + 3x^2 - 1$. Then use the graph to describe the increasing and decreasing behavior of the function.




increasing: $(-\infty, -2) \cup (0, \infty)$

decreasing: $(-2, 0)$

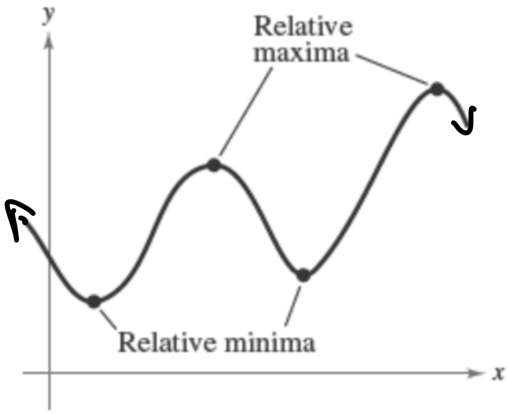
## Relative (local) maxima/minima

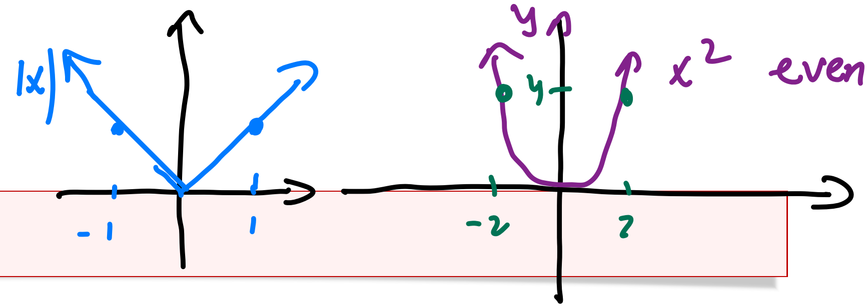



For a function $f$:

- A point is a **local maximum** if $f(x)$ is greater at that point than at nearby points.

- A point is a **local minimum** if $f(x)$ is less at that point than at nearby points




## Even/odd functions

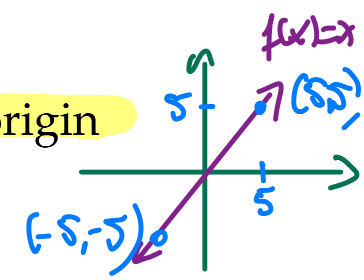
- A function is **even** if its graph is symmetric about the y-axis

$f(1)=1$
$f(-1)=1$

$$f(-x) = f(x) \text{ for all } x \text{ in the domain}$$

$f(-1) = f(1)$

- A function is **odd** if its graph is symmetric about the origin




$$f(-x) = -f(x) \text{ for all } x \text{ in the domain}$$

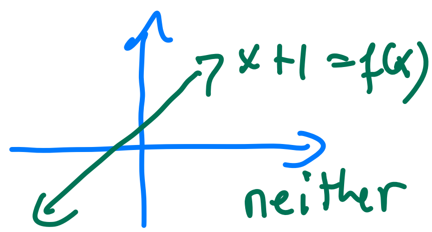
$f(-5) = -f(5)$

$-5 = -5$

**Example 2.** Determine whether the function is even, odd or neither.

1. $f(x) = x^3 + 4x^1$
2. $g(x) = 3x - x^2$
3. $f(t) = 5t^{2/3}$

$(-x)^3 = \underbrace{(-x)(-x)}_{x^2}(-x)$




1) $f(-x) = (-x)^3 + 4(-x) = -x^3 - 4x = -(x^3+4x) = -f(x)$

odd

2) $g(-x) = 3(-x) - (-x)^2 = \underbrace{-3x - x^2}_{\text{this is not}}$ neither

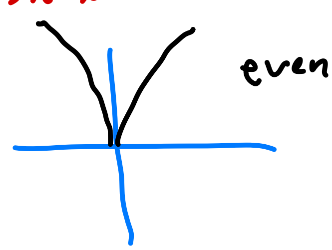
this is not $g(x)$ or $-g(x)$

$3x - x^2$ $\quad -3x + x^2$

3) $f(-t) = 5(-t)^{2/3} = 5\sqrt[3]{(-t)^2} = 5\sqrt[3]{t^2}$

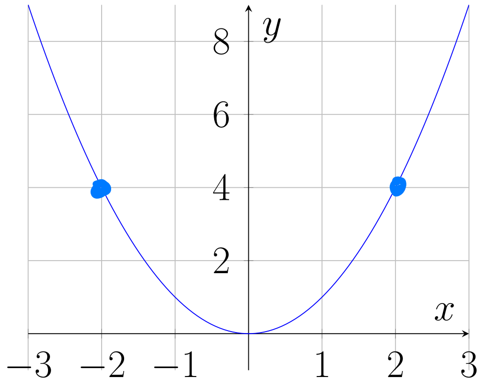
$= 5t^{2/3} = f(t)$

even



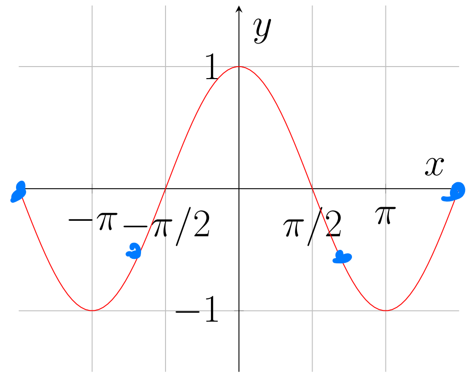
even

Function: $f_1(x) = x^2$



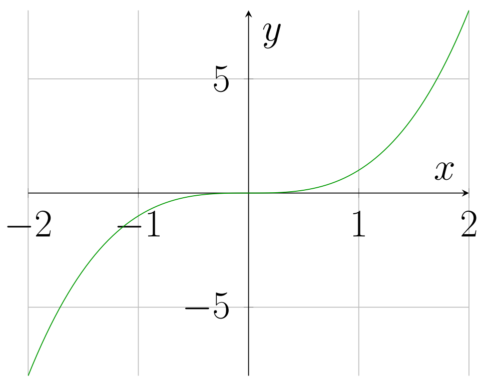

Function: $f_2(x) = \cos(x)$

even



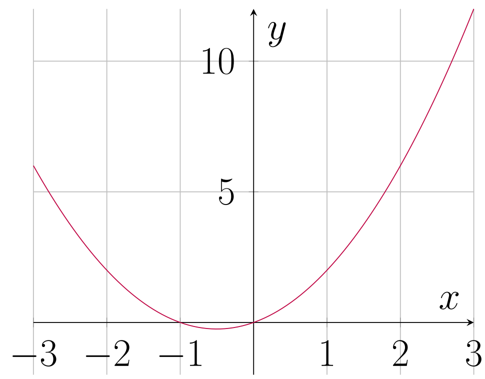

odd

Function: $f_3(x) = x^3$




$f_4(x) = x^2 + x$

neither




## Properties

- For even functions ($f_1$ and $f_2$), $f(x) = f(-x)$ for all $x$ in the domain
- For the odd function ($f_3$), $f(-x) = -f(x)$ for all $x$ in the domain
- For $f_4$, neither property holds: $f_4(-x) \neq f_4(x)$ and $f_4(-x) \neq -f_4(x)$