Tuesday, April 29th

Exam 4 on Thursday

~~$\log_e e$~~$^2 =$

EX Use the properties of natural logarithms to rewrite the expression, without using a calculator.

1) $\ln e^2 =$ 2 3) $\ln e =$ 1 $\log_e e = 1$

2) $5 \ln 1 =$ $5 \cdot 0 = 0$ 4) $e^{\ln(4.2)} =$ 4.2

Example 8. Find the domain in interval notation.

$y = \log_{1/2}(7 - 3x)$

$\log x$
To find domain
$x > 0$

To find domain solve $7 - 3x > 0$

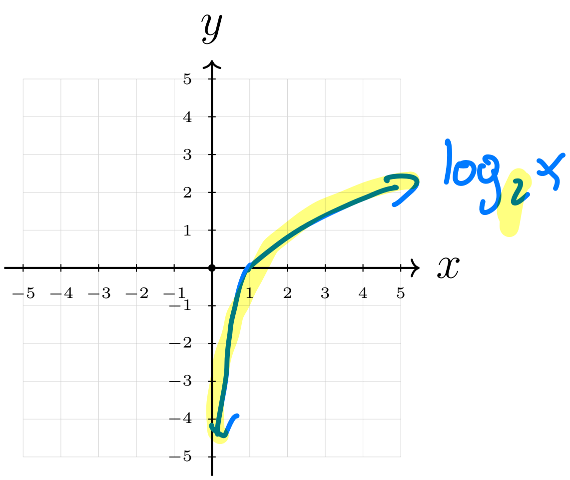
$-7 \quad -7$

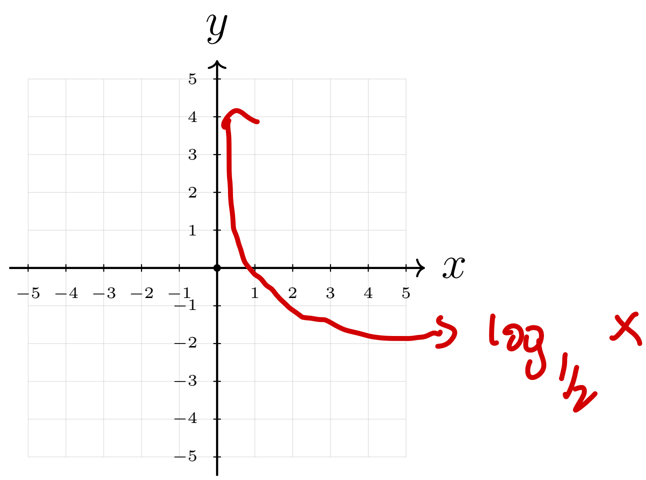
$-3x > -7$ / $\div (-3)$

$x < 7/3$

Domain: $(-\infty, 7/3)$

Graphs of logarithmic functions





increasing

$b > 1$

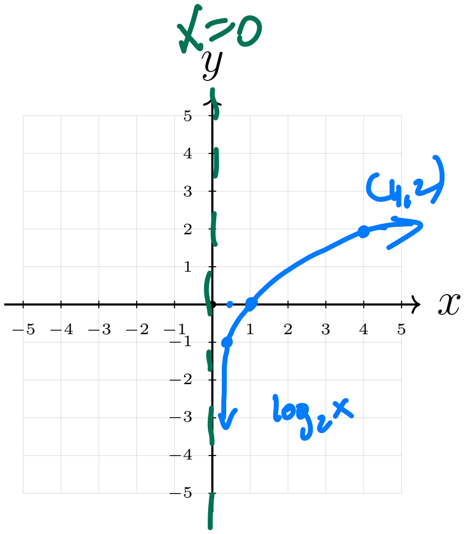
$0 < b < 1$

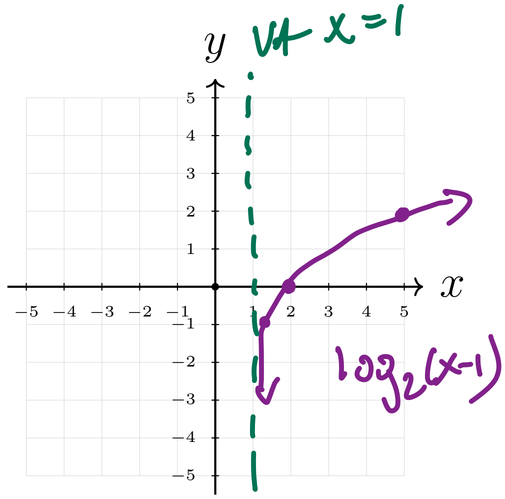
decreasing

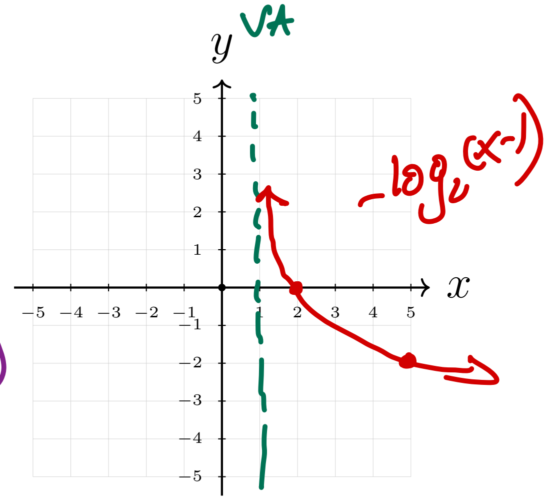
Asymptote: VA: $x = 0$ (y axis)

Example 9. Use the graph of the parent function to describe the transformation that yields the graph of g. Then find the domain and range in interval notation, the vertical asymptote and the x-intercept.

$g(x) = -\log_2(x-1)$







parent function

$\log_2 x$

$\log_2 4 = ?$

$\log_2 4 = 2$

bc

$2^2 = 4$

$\log_2 \frac{1}{2} = -1$

$2^{-1} = \frac{1}{2}$

$\log_2(x-1)$

right one unit

$-\log_2(x-1)$

reflect over x axis

Domain: $(1, \infty)$

Range: $(-\infty, \infty)$

x int : $(2, 0)$

Section 4.4 Laws/Properties of Logs

Properties of Logarithms with base b, where $b, x, y > 0, b \neq 1$, and p is a real number

$2^x \cdot 2^y = 2^{x+y}$

1. $\log_b 1 = 0$ — 5) $\log_b(x \cdot y) = \log_b x + \log_b y$ — product property
2. $\log_b b = 1$ — 6) $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$ — quotient property
3. $\log_b b^x = x$ — 7) $\log_b(x^p) = p \cdot \log_b x$ — power property
4. $b^{\log_b x} = x$

$2 \log x^2$

$\frac{2^x}{2^y} = 2^{x-y}$

Condensing Logarithmic Expressions

Write each expression as a single logarithm.

1. $\log_3(4) + \log_3(5)$ $\overset{(5)}{=} \log_3 4 \cdot 5 = \log_3 20$
2. $\log_2(9) - \log_2(3)$ $\overset{(6)}{=} \log_2 \frac{9}{3} = \log_2 3$
3. $2\log(x) - \log(y) + \log(z)$ $\overset{7}{=} \log x^2 - \log(y) + \log(z) = \log \frac{x^2}{y} + \log z$
   $= \log\left(\frac{x^2}{y} \cdot z\right)$
4. $\log_5(3) + \log_5(x^2) - \log_5(7)$ $= \log_5 3x^2 - \log_5 7 = \log_5 \frac{3x^2}{7}$
5. $\frac{1}{2}\log_6(x) + 3\log_6(y) - \log_6(z^2)$
6. $\log_b(x^m y^n) - \log_b(z^p) + \frac{1}{q}\log_b(w^r)$

(3) $\log x^2 \underbrace{- \log y + \log z}$ $= \log x^2 + (\log z - \log y)$
$= \log x^2 + \log \frac{z}{y}$
$= \log \frac{x^2 \cdot z}{y}$

5. $\frac{1}{2}\log_6(x) + 3\log_6(y) - \log_6(z^2)$

6. $\log_b(x^m y^n) - \log_b(z^p) + \frac{1}{q}\log_b(w^r)$

5) $\log_6 x^{1/2} + \log_6 y^3 - \log_6 z^2$

$= \log_6 x^{1/2} \cdot y^3 - \log_6 z^2$

$= \log_6 \frac{x^{1/2} y^3}{z^2} = \log_6 \frac{\sqrt{x}\, y^3}{z^2}$

6) $\log_b x^m y^n - \log_b z^p + \log_b w^{r\frac{1}{q}}$

$\log_b \frac{x^m y^n}{z^p} + \log_b w^{\frac{r}{q}} \rightarrow \sqrt[q]{w^r}$

$\log_b \left(\frac{x^m y^n}{z^p} \cdot \sqrt[q]{w^r} \right)$

Expanding Logarithmic Expressions

Write each expression as a sum or difference of logarithms.

1. $\log_4(12) = \log_4 3\cdot 4 = \log_4 3 + \log_4 4$
   $= \log_4 3 + 1$

2. $\log_7\left(\frac{a}{b}\right) = \log_7 a - \log_7 b$

3. $\log_3(5x^2y) = \log_3 5x^2 + \log_3 y$
   $= \log_3 5 + \log_3 x^2 + \log_3 y$
   $= \log_3 5 + 2\log_3 x + \log_3 y$

4. $\log_5\left(\frac{2^3x^4}{y^2z}\right) = \log_5 2^3x^4 - \log_5 y^2 z$
   $= \log_5 2^3 + \log_5 x^4 - (\log_5 y^2 + \log_5 z)$
   $= 3\log_5 2 + 4\log_5 x - 2\log_5 y - \log_5 z$

5. $\log\left(\frac{(ab)^3\sqrt[3]{c}}{\sqrt{d^5}}\right)$

6. $\log_b\left(\sqrt[n]{\frac{x^my^p}{z^qw^r}}\right) = \log_b\left(\frac{x^my^p}{z^pw^p}\right)^{\frac{1}{n}}$
   $= \frac{1}{n}\log_b \frac{x^my^p}{z^pw^p} =$
   $= \frac{1}{n}\left(\log_b x^m y^p - \log_b z^p w^p\right)$
   $= \frac{1}{n}\left(\log_b x^m + \log_b y^p - (\log_b z^p + \log_b w^p\right.$

~~Section~~ Solving Exponential and Logarithmic Equations

1. $5^{2x} = 3$
   $\log 5^{2x} = \log 3$
   $2x\cdot\log 5 = \log 3$
   $x = \frac{\log 3}{2\log 5}$ exact answer
   $x \approx 0.314$

2. $160e^{-0.03t} = 80$
   $\frac{160e^{-0.03t}}{160} = \frac{80}{160}$
   $e^{-0.03t} = 1/2$
   $\ln e^{-0.03t} = \ln 1/2$
   $-0.03t \ln e = \ln 1/2$
   $-0.03t = \ln 1/2$
   $t = \frac{\ln(1/2)}{-0.03}$

3. $96 - 10^x = 12$

4. $e^{x^2+x} = e^{3x+15}$

5. $(1 + 0.05/4)^{4t} = 3$

Solving Exponential Equations

- isolate the $\boxed{\text{exponential expression}}$ ✓
- take the log of both sides
- simplify using log properties
- solve the equation by isolating the $\boxed{\text{variable}}$

4. $e^{x^2+x} = e^{3x+15}$

$e^2 = e^x$

$x = 2$

5. $(1 + 0.05/4)^{4t} = 3$

4) $x^2 + x = 3x + 15$

$x^2 + x - 3x - 15 = 0$

$x^2 - 2x - 15 = 0$

$(x+3)(x-5) = 0$

$x = -3$ ✓ $x = 5$ ✓

Check: $e^{(-3)^2+(-3)} \stackrel{?}{=} e^{3(-3)+15}$

$e^{9-3} \stackrel{?}{=} e^{-9+15}$

$e^6 = e^6$ ✓

$e^{25+5} \stackrel{?}{=} e^{15+15}$

$e^{30} = e^{30}$

5) $\log (1 + 0.05/4)^{4t} = \log 3$

$$\frac{\cancel{4}t \log \cancel{(1+0.05/4)}}{\cancel{4} \log \cancel{(1+0.05/4)}} = \frac{\log 3}{4 \log (1+0.05/4)}$$

$$t = \frac{\log 3}{4 \log (1+0.05/4)}$$

$$\log\left(\frac{(ab)^3\sqrt[3]{c}}{\sqrt{d^5}}\right) = \log (ab)^3 \cdot \sqrt[3]{c} - \log \sqrt{d^5}$$

$$= \log (ab)^3 + \log \sqrt[3]{c} - \log \sqrt[2]{d^5}$$

$$= 3\log ab + \log c^{1/3} - \log d^{5/2}$$

$$= 3\,\underbrace{\log ab} + \frac{1}{3}\log c - \frac{5}{2}\log d$$

$$= 3\log a + 3\log b + \frac{1}{3}\log c - \frac{5}{2}\log d$$

EX Solve the logarithmic equation algebraically.

$x = ?$ $t = ?$

1. $\log(2x+4) = \log(3-x)$

$2x+4 = 3-x$
$2x+x = -1$
$3x = -1$
$x = -1/3$ ✓

$\log(2(-\frac{1}{3})+4) \stackrel{?}{=} \log 3-(\frac{-1}{3})$
$\log(-\frac{2}{3}+4) \stackrel{?}{=} \log 3+\frac{1}{3}$
$\log(-\frac{2}{3}+\frac{12}{3})$ ✓
$\log(\frac{10}{3}) = \log\frac{10}{3}$

2. $\ln(x) + \ln(x-3) = \ln(5x-7)$

$\ln x(x-3) = \ln(5x-7)$
$x(x-3) = 5x-7$
$x^2-3x-5x+7 = 0$
$x^2-8x+7 = 0$

$(x-7)(x-1) = 0$
$x = 7$ & $\cancel{x = 1}$

$\ln 7 + \ln(7-3) \stackrel{?}{=} \ln(5\cdot 7-7)$
$\ln 7 + \ln 4 \stackrel{?}{=} \ln 28$
$\ln 7\cdot 4 = \ln 28$ ✓

$\ln 1 + \cancel{\ln(1-3)} = \ln(5\cdot 1-7)$

3. $2\log_5 x = \log_5(7) + \log_5(3x)$

$\log_5 x^2 = \log_5 21x$
$\log_5 * = \log_5 ☆$

$x^2 = 21x$
$x^2-21x = 0$
$x(x-21) = 0$
$\cancel{x = 0}$ $x = 21$ ✓

Solving log equations (all terms have a log with the same base)

✓1. condense logs into a single log (first use prop 7), if multiple logs on one side

2. solve the equation using the arguments

3. check and discard extraneous solutions !!!

4. $-3+4\ln(7x)=2$ $\quad /+3$

$4\ln 7x = 5$

$\ln (7x)^4 = 5$

$e^5 = (7x)^4$

write it in exponential form

$e^5 = (7x)^4 \quad /\sqrt[4]{\ }$

$\sqrt[4]{e^5} = \sqrt[\cancel{4}]{(7x)^{\cancel{4}}}$

$\sqrt[4]{e^5} = 7x \quad /\div 7$

$\frac{\sqrt[4]{e^5}}{7} = x$

check $x = \frac{\sqrt[4]{e^5}}{7}$

$-3+4\ln \cancel{7}\cdot\frac{\sqrt[4]{e^5}}{\cancel{7}} =$

$= -3+4\ln e^{5/4}$

$= -3+\cancel{4}\cdot\frac{5}{\cancel{4}} = 2$

5. $\ln(\sqrt{x-8})=5$

$e^5 = \sqrt{x-8} \quad /^\wedge 2$

$e^{10} = x-8 \Rightarrow x = e^{10}+8$

$\ln\sqrt{x-8} = 5$

$\ln(x-8)^{1/2} = 5$

$\frac{1}{2}\ln(x-8) = 5 \quad /\cdot 2$

$\ln(x-8) = 10$

$e^{10} = x-8$

6. $\frac{3\log_2(x+4)}{3}=\frac{7}{3}$

$\log_2(x+4) = 7/3$

$2^{7/3} = x+4 \quad /-4$

$x = 2^{7/3}-4$

7. $\log_4(x+2)=1-\cancel{\log_4(x-1)}$

$+\log_4(x-1) \qquad + \cancel{\log_4(x-1)}$

$\log_4(x+2)+\log_4(x-1)=1$

$\log_4(x+2)(x-1)=1$

$4^1 = (x+2)(x-1)$

$(x+2)(x-1)=4$

$x^2-x+2x-2-4=0$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3 \qquad x=2$ ✓

8. $\log x = 2+\log(x-3)$

Solving log equations (has a term without a log)

1. isolate terms with a log of same base to one side
2. condense logs into a single log and isolate
3. write as an exponential equation
4. solve the equation
5. check and discard extraneous solutions

I

$$\log x = 2 + \log(x-3)$$
$$-\log(x-3) \qquad -\log(x-3)$$

II

$$\underbrace{\log x - \log(x-3)}_{} = 2$$

$$\log_{10} \frac{x}{x-3} = 2$$

III

$$10^{2} = \frac{x}{x-3}$$

$$\frac{100}{1} = \frac{x}{x-3}$$

$$100(x-3) = x$$

$$100x - 300 = x$$

$$99x - 300 = 0$$

$$99x = 300$$

$$x = \frac{300}{99} = \frac{100}{33}$$

$$\frac{\cancel{x}}{\cancel{x}-3}$$

$$\frac{\cancel{x}}{\cancel{x}-3} =$$