

Section 4.6 Graphs of Other Trig Functions

The Tangent Function

Library of Parent Functions: Tangent Function

Recall:

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cos x \neq 0$$

$$x \neq \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}$$

The basic characteristics of the parent tangent function are summarized below.

Domain:

Range: $(-\infty, \infty)$

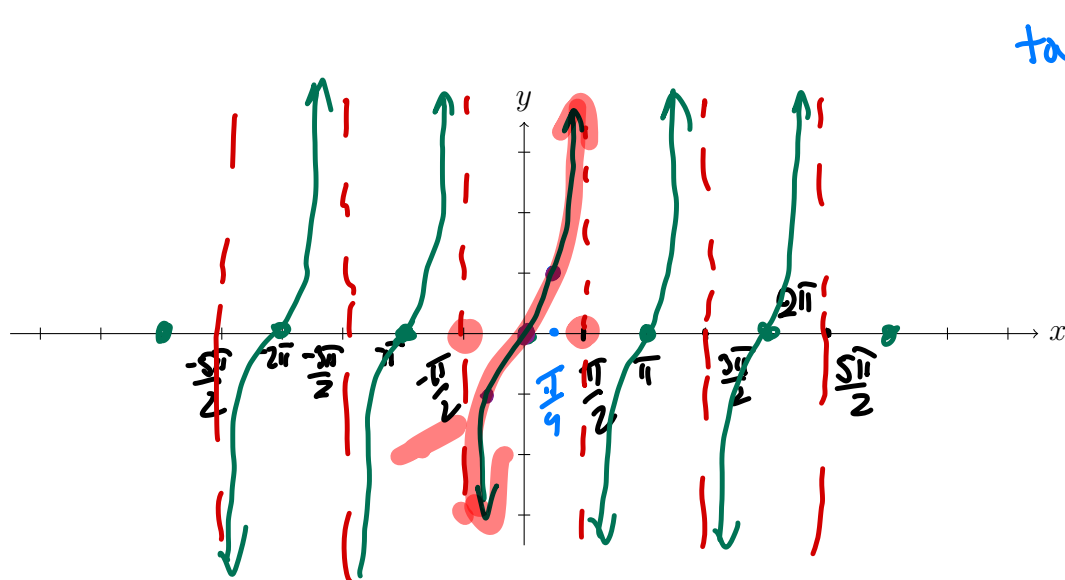
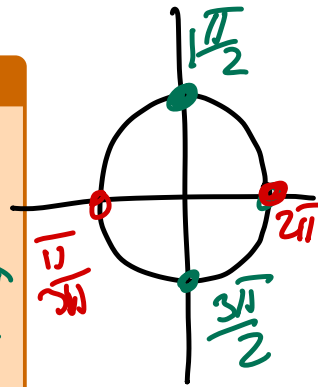
Period: π

x -intercepts: $n\pi, n \text{ integer}$

y -intercept: $(0, 0)$

Vertical asymptotes:

Odd function : origin symmetry



$$\tan \frac{\pi}{4} = 1$$

x intercepts : $\tan x = \frac{\sin x}{\cos x} = 0$ when $\sin x = 0$ for multiples of π
 $n\pi, n \text{ integer}$

$$y \text{ intercept : } \tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$$

Sketching the graph of $y = a \tan(bx - c)$ is similar to sketching the graph of $y = a \sin(bx - c)$ in that you locate key points that identify the intercepts and asymptotes. Two consecutive asymptotes can be found by solving the equations $bx - c = -\pi/2$ and $bx - c = \pi/2$ for x . The midpoint between two consecutive asymptotes is an x -intercept of the graph. The period of the function $y = a \tan(bx - c)$ is the distance between two consecutive asymptotes. The amplitude of a tangent function is not defined. After plotting the asymptotes and the x -intercept, plot a few additional points between the two asymptotes and sketch one cycle. Finally, sketch one or two additional cycles to the left and right.

$$\text{period} = \pi/b$$

Example 1 Library of Parent Functions: $f(x) = \tan x$

Sketch the graph of $y = \tan \frac{x}{2}$ by hand.

$$y = a \tan(bx - c)$$

Solution:

$$a = 1, b = \frac{1}{2}, c = 0$$

Endpoints:

$$\text{Left: } bx - c = -\frac{\pi}{2}$$

$$\frac{1}{2}x - 0 = -\frac{\pi}{2}$$

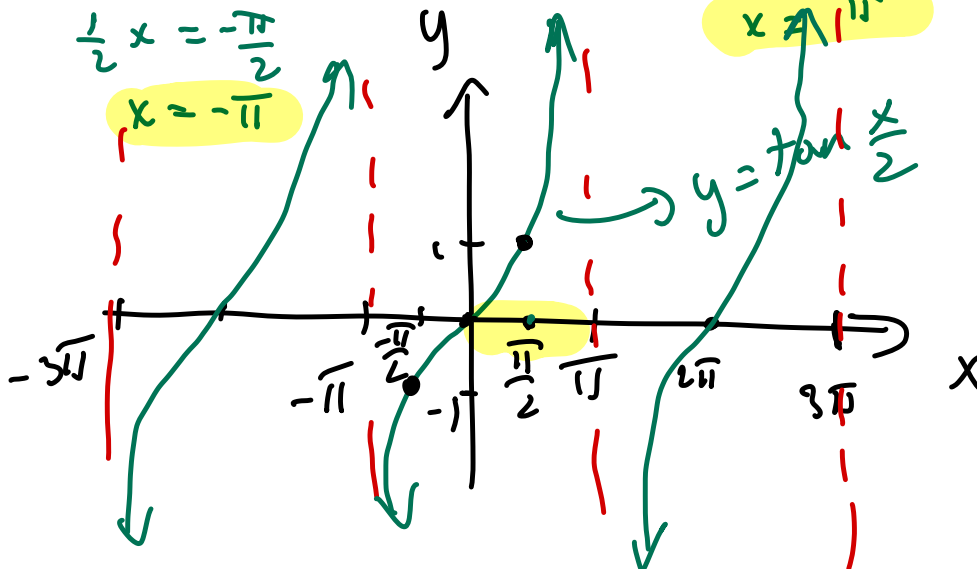
$$\frac{1}{2}x = -\frac{\pi}{2}$$

$$x = -\pi$$

$$\text{Right } bx - c = \frac{\pi}{2}$$

$$2 \cdot \frac{1}{2}x = \frac{\pi}{2} \cdot 2$$

$$x = \pi$$



$$\tan \frac{\pi/2}{2} = \tan \frac{\pi}{4} = 1$$

$$x = \frac{\pi}{2}$$

$$\tan \frac{x}{2} = \tan \frac{\pi/2}{2} = \tan \frac{\pi}{4}$$

Example 2 Library of Parent Functions: $f(x) = \tan x$

Sketch the graph of $y = -3 \tan(2x)$ by hand.

$$a \tan(bx - c)$$

Solution:

$$a = -3$$

$$b = 2$$

$$c = 0$$

$$\text{period} = \frac{\pi}{b} = \frac{\pi}{2}$$

Left:

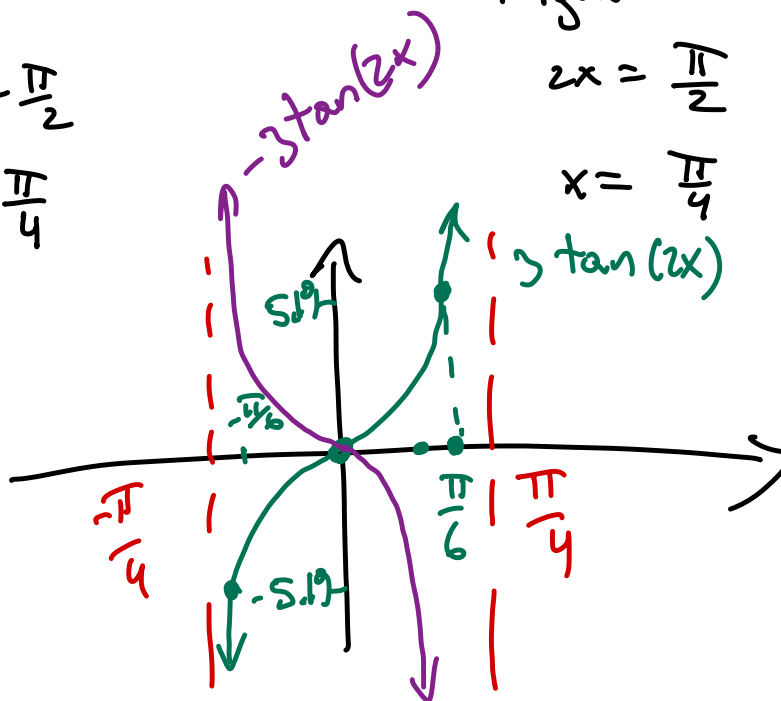
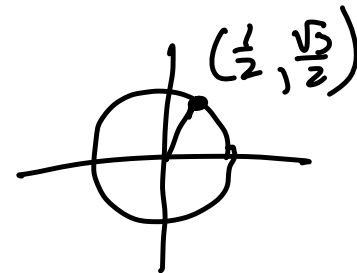
$$2x = -\frac{\pi}{2}$$

$$x = -\frac{\pi}{4}$$

Right:

$$2x = \frac{\pi}{2}$$

$$x = \frac{\pi}{4}$$



$$x = \frac{\pi}{6}$$

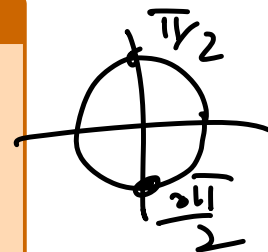
$$3 \tan\left(2 \cdot \frac{\pi}{6}\right) = 3 \tan \frac{\pi}{3} = 3\sqrt{3} = 5.19$$

The Cotangent Function

Library of Parent Functions ^{Cot} Tangent Function

Recall:

$$\cot x = \frac{\cos x}{\sin x}, \quad \sin x \neq 0$$



The basic characteristics of the parent tangent function are summarized below.

Domain:

Range:

Period:

x -intercepts:

y -intercept:

Vertical asymptotes:

Odd function

$x \neq n\pi$ (all real number except multiples of π)

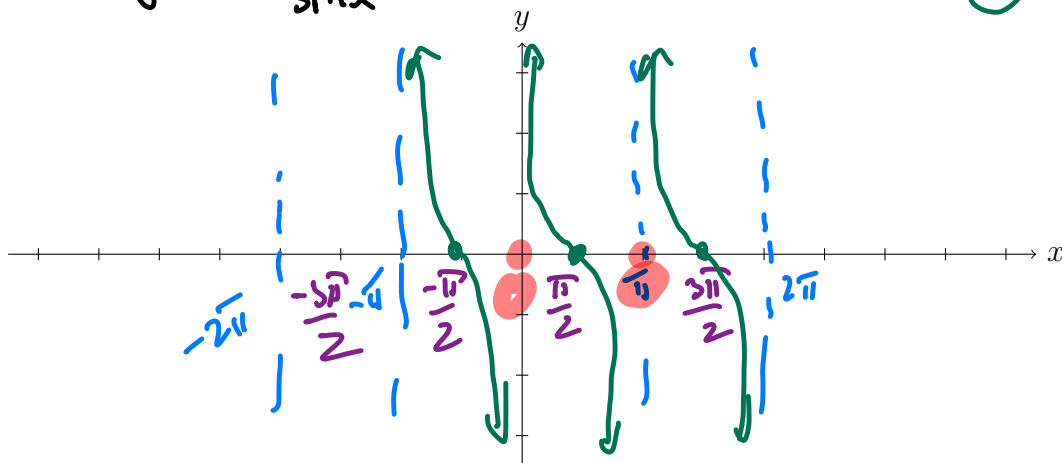
$(-\infty, \infty)$

π

where $\cos x = 0$, $x = \pm\pi/2, \pm3\pi/2, \pm5\pi/2, \dots$

y -intercept: set $x=0$ $\cot 0 = \frac{\cos 0}{\sin 0} = \frac{1}{0}$ undefined (no y -intercept)

x -intercepts: set $y=0$ $\frac{\cos x}{\sin x} = 0 \Rightarrow \cos x = 0, \pm\pi/2, \pm3\pi/2, \dots$



Example 3 Library of Parent Functions: $f(x) = \cot x$

Sketch the graph of $y = 2 \cot \frac{x}{3}$ by hand.

$$y = a \cot(bx - c)$$

Solution:

$$\begin{aligned} a &= 2 \\ b &= \frac{1}{3} \\ c &= 0 \end{aligned}$$

$$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{1/3} = 6\pi$$

Left

$$bx - c = 0$$

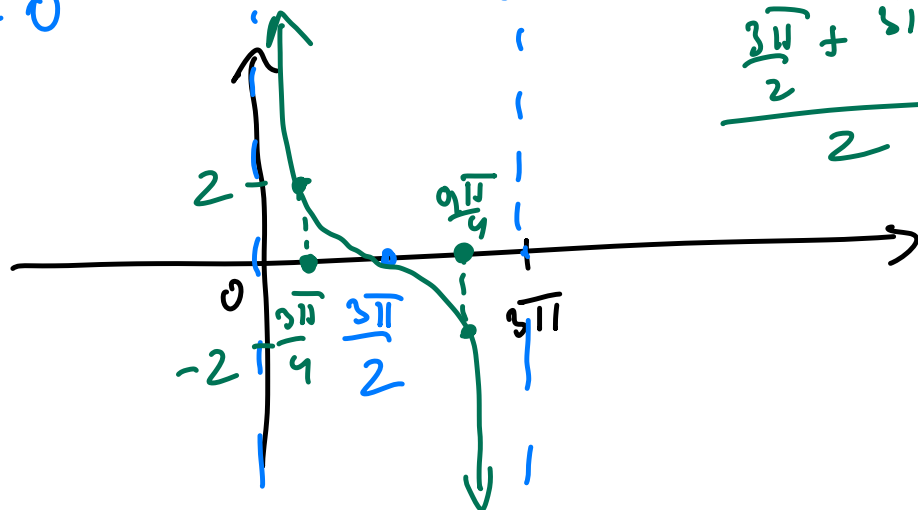
$$\frac{x}{3} = 0$$

$$x = 0$$

$$bx - c = \pi$$

$$\frac{x}{3} = \pi$$

$$x = 3\pi$$

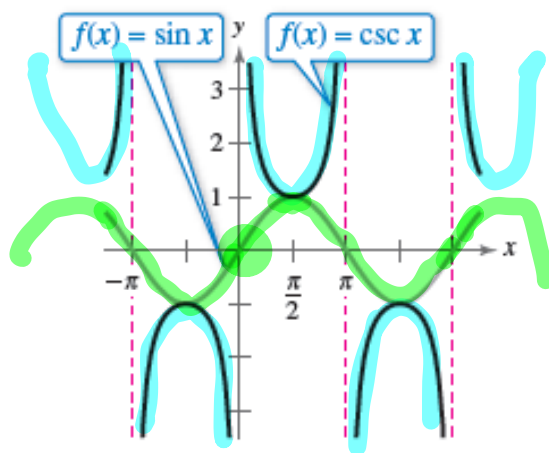


$$\begin{aligned} \frac{\frac{3\pi}{2} + 3\pi}{2} &= \frac{\frac{3\pi}{2} + \frac{6\pi}{2}}{2} \\ &= \frac{9\pi}{4} \end{aligned}$$

Graphs of Reciprocal functions

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$



Domain: all real numbers x , $x \neq n\pi$

Range: $(-\infty, -1] \cup [1, \infty)$

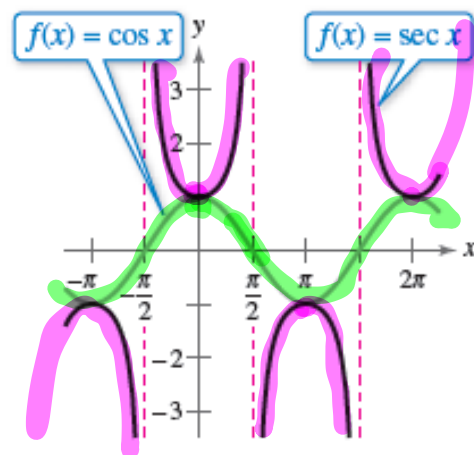
Period: 2π

No intercepts

Vertical asymptotes: $x = n\pi$

Odd function

Origin symmetry



Domain: all real numbers x , $x \neq \frac{\pi}{2} + n\pi$

Range: $(-\infty, -1] \cup [1, \infty)$

Period: 2π

y-intercept: $(0, 1)$

Vertical asymptotes: $x = \frac{\pi}{2} + n\pi$

Even function

y-axis symmetry