Math 140: Thursday, Oct 31, Day 17 Notes

Agenda for Thursday, Oct 31, 2024	
Finish Probability !!!	**1**

Coming up

- ! Online HW 10 due Tuesday and Online HW 11 posted today
- Breakfast at Tutoring - Monday, November 4th,8 am - 11 am, Grayslake Tutoring Center (L135)

Binomial Distribution

Next, we will look into a special discrete probability distributions called **Binomial Distributions.**
There are **four conditions** that the experiment has to meet to be considered a **binomial experiment:** ex: tossing a coin $n=20$ times

- Condition 1: set amount of trials. We denote the number of trials with n.
- Condition 2: the outcome of each trial has only 2 possibilitie : "success", "failure"
- Condition 3: each trial is independent
- Condition 4: the probability of success, denoted p stays the same from trial to trial.

Example 1. Are the following binomial experiment:

a) Randomly guessing at a multiple-choice question with 4 possible answers has only two outcomes. If a success is guessing correctly, then a failure is guessing incorrectly. Suppose there are 6 multiple choice questions. Joe guesses on each question with no pattern.

- Condition 1: $n = 6$
- Condition 2: ✓
- Condition 3: ✓
- Condition 4: $p = 1/4 = 0.25$

yes

b) Randomly guessing at a multiple-choice question has only two outcomes. If a success is guessing correctly, then a failure is guessing incorrectly. Suppose there are 6 multiple choice questions. The first 3 questions have 4 choices, and the last 3 questions have 5 choices each. Cesar guesses on each question with no pattern. Determine if this is a binomial experiment.

- Condition 1:
- Condition 2:
- Condition 3:
- Condition 4: ✗

Q_{1-3}: a, b, c, d, $p = 1/4$

NO

Q_{4-6}: a, b, c, d, e, $p = 1/5$

c) Cyanosis is the condition of having bluish skin due to insufficient oxygen in the blood. About 80% of babies born with cyanosis recover fully. A hospital is caring for 10 babies born with cyanosis. The random variable represents the number of babies that recover fully.

- Condition 1: $n = 10$
- Condition 2: ✓
- Condition 3: ✓
- Condition 4: $p = .8$

yes

Notation and Properties of the Binomial distribution

- The outcomes of a binomial experiment fit a binomial probability distribution.
- We say the random variable $X =$ # of successes in n trials is a binomial random variable and we write means X is binomial

  $X \sim B(n, p)$

- To find the probability that our random variable X takes on a certain value (denoted k) we use the following formula:

  $P(x = k) =$ $\underbrace{\textbf{binomialpdf(n, p, k)}}_{\text{calculator}}$ Ex: $X \sim B(10, .8)$

  ex: $P(X=7) = \text{binomialpdf}(10, .8, 7) = 0.201$

- To find the probability that our random variable X takes on valeus less than or equal to a certain value (denoted k) we use the following formula:

  $P(x \leq k) =$ **binomialcdf(n, p, k)**

  c → cumulative

  $P(x \leq 7) = \text{binomialcdf}(10, .8, 7) = .322$

- The **mean** of a binomial probability distribution is

  mean → average or expected value

  $\mu = np$ $\mu = (10)(.8) = 8$ babies

Example 2. Randomly guessing at a multiple-choice question with 4 possible answers has only two outcomes. If a success is guessing correctly, then a failure is guessing incorrectly. Suppose there are 6 multiple choice questions. Mayra guesses on each question with no pattern. $n = 6$ $p = 0.25$

a) Find the probability of Mayra guessing correctly exactly 3 questions.

$P(x=3) = \text{binomialpdf}(6, 0.25, 3) = .132$

b) Find the probability of Mayra guessing correctly all of the questions.

$P(x=6) = \text{binomialpdf}(6, 0.25, 6) = .000244$

c) Find the mean or expected number of questions Mayra guesses correctly.

$\mu = np = (6)(.25) = 1.5$ questio

Example 3. It has been stated that about 41% of adult workers have a high school diploma but do not pursue any further education. 20 adult workers are randomly selected. $n = 20, \ p = .41$

a) Find the probability that **exactly** 2 of them have a high school diploma but do not pursue any further education.

$P(X=2) = \text{binomialpdf}(20, .41, 2) =$

b) Find the probability that **at most** 12 of them have a high school diploma.

$P(X \le 12) = \text{binomialcdf}(20, .41, 12)$

c) Find the probability that **at least** 6 of them have a high school diploma.

$P(X \ge 6) = 1 - \text{binomialcdf}(20, .41, 5)$

d) Find the probability that **more than** 10 of them have a high school diploma.

$P(X > 10) = 1 - \text{binomialcdf}(20, .41, 10)$

e) Find the probability that **less** 5 of them have a high school diploma.

$P(X < 5) = \text{binomialcdf}(20, .41, 4)$

f) How many adult workers do you **expect** to have a high school diploma but do not pursue any further education?

$\mu = np = (20)(.41) = 8.2$ people