Section 1.3 Hamiltonian Circuits and Paths

HP

Hamiltonian Path - A path that visits each vertex exactly once.

HC Hamiltonian circuit - A circuit in a graph is a path that starts and ends at the same vertex and passes through all other vertices exactly once.

Question: What is the difference between a Hamilton Path and an Euler Path?

HP: visits each vertex exactly once.
Some edges may not be used.

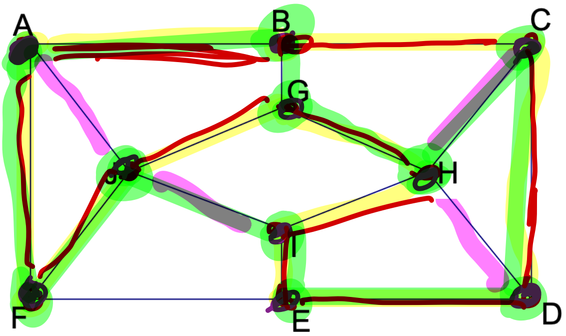
EP: travels each edge exactly once.
some vertices may be used more than once.

Notes:

even degrees! 2 odd degree!

1. A graph can't have both an Euler circuit and an Euler path.
2. A graph that has a Hamilton Circuit has a Hamilton Path.

Example 1. Consider the following graph.



1. Find two different Hamilton Circuits.

- HC that starts and ends @ vertex A.

* ABCDEIHGJFA

BCDEIHGJFAB → is considered the same circuit

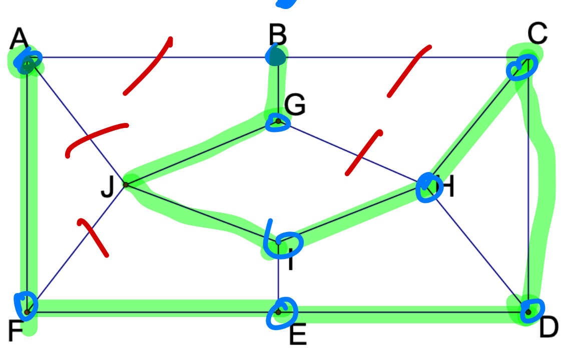
- HC that starts @ vertex B

BGHCDEIJFAB

2. Find a Hamilton Path that starts at A and ends at B.

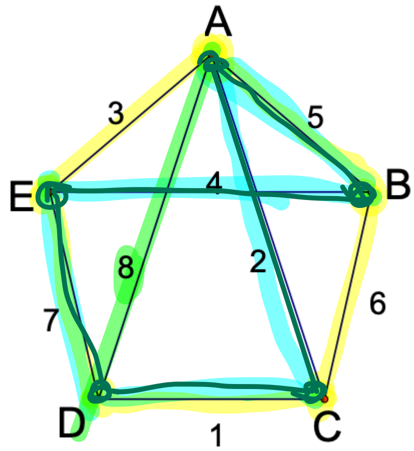
start A

end



AFEDCHIJGB

Example 2. Consider the following weighted graph.



a) What is the weight of AD? 8

b) What is the weight of the Hamilton Circuit ABCDEA?

$5+6+1+7+3 = 22$

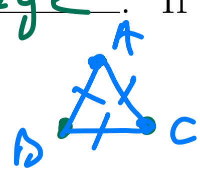
c) What is the weight of the Hamilton Circuit ACDEBA?

$2+1+7+4+5 = 19$

Note: A graph has many HC.

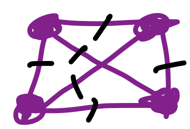
Complete graph A graph in which every pair of distinct vertices is joined by an edge. If you have N vertices, K_N denotes the complete graph.

$N=3$



K_3 has 3 edges

$N=4$



K_4

K_4 has 6 edges

Factorial:

we use ! to mean factorial.

$2! = 1 \cdot 2 = 2$

read 2 factorial

$4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$

$N! = 1 \cdot 2 \cdot 3 \cdot 4 \cdots N$

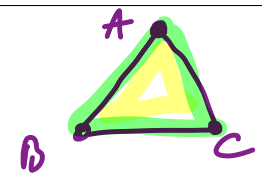
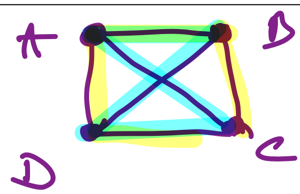
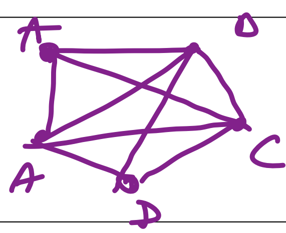
$\boxed{0! = 1}$

! Formula for the number of edges in a complete graph with N vertices:

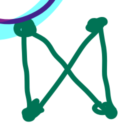
K_N has $\boxed{\frac{N(N-1)}{2}}$ edges. ex K_4 has $\frac{4(4-1)}{2} = \frac{4\cdot 3}{2} = 6$ edges.

Table with info about complete graphs

Table 1: Complete Graphs Facts

No vertices	Notation	Visualization of Complete graph	Number of HCs
$N=3$	K_3		ABCA ACBA 2 HCs
$N=4$	K_4		ABCDA, ADCBA ACDBA, ABDCA ADBCA, ACBDA
$N=5$	K_5		K_5 has 24 HCs

6 HCs



! Formula for the number of Hamilton Circuits in the complete graph K_N: is $(N-1)!$

$N=3$: K_3 has $(3-1)! = 2! = 2$

$N=4$: K_4 has $(4-1)! = 3! = 1\cdot 2\cdot 3 = 6$

$N=5$ K_5 has $(5-1)! = 4! = 1\cdot 2\cdot 3\cdot 4 = 24$

Example 3. For a complete graph with 10 vertices, how many Hamilton Circuits are there?

$N=10$

K_{10} has $(10-1)! = 9! = 1\cdot 2\cdot 3 \cdots 9 = 362,880$ HCs

Example 4. If K_N has 40,320 Hamilton Circuits, what is N?

K_{10} has $\frac{10(10-1)}{2} = \frac{10\cdot 9}{2} = 45$ edges

$N=?$

st K_N has 40320 HC

$40,320 = 8! \Rightarrow (9-1)!$ this gives $N=9$ K_9