

Setion 5.1: Using Fundamental Trigonometric Identities

1 Recall: Fundamental Trigonometric Identities

Type	Identities
Reciprocal	$\sin \theta = \frac{1}{\csc \theta}$ $\cos \theta = \frac{1}{\sec \theta}$ $\tan \theta = \frac{1}{\cot \theta}$ $\csc \theta = \frac{1}{\sin \theta}$ $\sec \theta = \frac{1}{\cos \theta}$ $\cot \theta = \frac{1}{\tan \theta}$
Quotient	$\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\cot \theta = \frac{\cos \theta}{\sin \theta}$
Pythagorean	$\sin^2 \theta + \cos^2 \theta = 1$ $1 + \tan^2 \theta = \sec^2 \theta$ $1 + \cot^2 \theta = \csc^2 \theta$
Cofunction	$\sin(\frac{\pi}{2} - \theta) = \cos \theta$ $\cos(\frac{\pi}{2} - \theta) = \sin \theta$ $\tan(\frac{\pi}{2} - \theta) = \cot \theta$ $\cot(\frac{\pi}{2} - \theta) = \tan \theta$ $\sec(\frac{\pi}{2} - \theta) = \csc \theta$ $\csc(\frac{\pi}{2} - \theta) = \sec \theta$
Even/Odd	$\sin(-\theta) = -\sin \theta$ (odd) $\cos(-\theta) = \cos \theta$ (even) $\tan(-\theta) = -\tan \theta$ (odd) $\cot(-\theta) = -\cot \theta$ (odd) $\sec(-\theta) = \sec \theta$ (even) $\csc(-\theta) = -\csc \theta$ (odd)

Table 1: Fundamental Trigonometric Identities

2 Example 1: Simplifying Trigonometric Expressions

2.1 Problem 1.1: Simplify $\sin x \cos^2 x - \sin x$

Solution:

2.2 Problem 1.2: Simplify $\sin t + \cot t \cos t$

Solution:

3 Example 2: Factoring Trigonometric Expressions

3.1 Problem 2.1: Factor $\sec^2 \theta - 1$

Solution:

3.2 Problem 2.2: Factor $4 \tan^2 \theta + \tan \theta - 3$

Solution:

3.3 Problem 2.3: Factor $\csc^2 x - \cot x - 3$

Solution:

4 Example 3: Rewriting Trigonometric Expressions

4.1 Problem: Rewrite $\frac{1}{1 + \sin x}$

Solution:

5 Example 4: Trigonometric Substitution

5.1 Problem: Use the substitution $x = 2 \tan \theta$ (with θ in quadrant I) to rewrite $\sqrt{4 + x^2}$

Solution: