

Mth 146: Day 2

7.1 Integration by Parts

1 Part 1: New Concepts & Worked Examples

Our next goal is to learn a method to solve integrals of the type $\int f(x)g(x) dx$ where there is no relationship between f and g , meaning one is not the derivative of the other. Examples: $\int x^2 e^x dx$, $\int x \arctan(x) dx$.

This will be accomplished with Integration by Parts (IBP) which is based on the reversal of the product rule.

Recall product rule:

$$(u(x) v(x))' = u'(x) v(x) + u(x) v'(x) \quad \left| \begin{array}{l} \text{integrate} \\ \text{both} \\ \text{sides} \end{array} \right.$$

$$\int (u(x) v(x))' dx = \int u'(x) v(x) + u(x) v'(x) dx$$

$$\int (u(x) v(x))' dx = \int u'(x) v(x) dx + \int u(x) v'(x) dx$$

solve for this one

$$3 = 5 - 2$$

$$\int u(x) v'(x) dx = \int (u(x) v(x))' dx - \int u'(x) v(x) dx$$

$$\int u(x) v'(x) dx = u(x) v(x) - \int u'(x) v(x) dx$$

$$\int u dv = uv - \int v du$$

$$\int x e^x dx = x \cdot e^x - \int e^x dx$$

$$= x e^x - e^x + C$$

differentiate

$$u = x \\ du = dx$$

integrate

$$dv = e^x dx \\ v = e^x$$

Integration by Parts Formula: For differentiable functions u, v , we have

$$\int u(x)v'(x) dx = u(x)v(x) - \int u'(x)v(x) dx$$

$$\int u dv = uv - \int v du.$$

Assuming that the relevant functions are defined on the interval $[a, b]$, calculating definite integrals can be done using integration by parts by the following formula:

Integration by Parts for Definite Integrals

$$\int_a^b u(x)v'(x) dx = u(x)v(x) \Big|_a^b - \int_a^b u'(x)v(x) dx.$$

Example 1. $\int x e^x dx$

solved on previous page

Example 2. $\int \theta \sec^2(2\theta) d\theta$

$$= \theta \frac{\tan(2\theta)}{2} - \int \frac{\tan(2\theta)}{2} d\theta$$

$$= \frac{1}{2} \theta \tan(2\theta) - \frac{1}{2} \frac{\ln|\sec 2\theta|}{2} + C$$

IBP (LIPET) ✓

$u = \theta$
 $du = 1 d\theta$

$dv = \sec^2(2\theta)$
 $v = \frac{\tan(2\theta)}{2}$

$\int u dv = \underline{uv} - \int v du$

$\int \tan x = \ln|\sec x|$

Example 3. $\int e^x \sin(x) dx$

How to choose u and dv wisely

L (logarithm)

I (inverses: $\arctan x$, $\arccos$

P (polynomials: x , x^2 , θ , θ^3 , t , ...)

E (exponentials: e^x , 5^x , e^{2x} , ...)

T (trig ($\sin x$, $\sec x$, $\tan x$))

Choose u in this order!!!