1. Rational Functions - Review

In this section we will review rational functions and concentrate on how to graph them by hand.

A rational function is a function of the form $f(x) = \dfrac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials. Even though rational fractions are constructed from polynomials, their graphs look quite different from the graphs of polynomial functions.

Examples of rational functions:
For each function, decide if it is a rational function and then if it is find its domain, x-intercepts and y-intercept.

1. $f(x) = \dfrac{x^2+3}{(x-1)(x+2)}$. yes

Domain: denom $\neq 0$
$x \neq 1$, $x \neq -2$
$(-\infty,-2) \cup (-2,1) \cup (1,\infty)$

x intercepts: $0 = \dfrac{x^2+3}{(x-1)(x+2)}$

$x^2+3=0$?
$x^2=-3$ NO xint

y intercepts: $f(0) = \dfrac{0^2+3}{(0-1)(0+2)} = \dfrac{3}{-2}$

2. $f(x) = \dfrac{x^2-3}{(x^2+1)(x+2)}$.

yes

Domain: $x \neq -2$, $x^2+1=0 \Rightarrow x^2=-1$ never 0.

y int: $f(0) = \dfrac{-3}{2}$
x int: $x^2-3=0 \Rightarrow x^2=3$
$\boxed{x = \pm\sqrt{3}}$

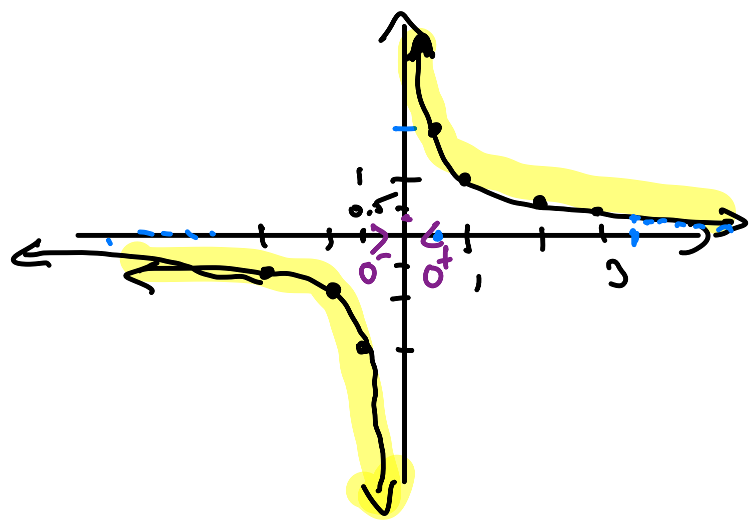
3. $f(x) = \dfrac{x^2+3}{(\sqrt{x}-1)(x+2)}$. NO

The **domain of a rational function** consists of all real numbers x except those for which the denominator is zero. When graphing a rational function, we must pay special attention to the behavior of the graph near those x-values.

2. A simple rational Function $f(x) = \frac{1}{x}$

Graph the rational function $f(x) = \frac{1}{x}$ and state its domain and range. $y \neq 0$

$x \neq 0$



x	f(x)
1	$\frac{1}{1} = 1$
2	$\frac{1}{2} = 0.5$
$\frac{1}{2}$	$\frac{1}{1/2} = 2$
3	$\frac{1}{3} = 0.\bar{3}$
-1	$\frac{1}{-1} = -1$
-2	$\frac{1}{-2} = -0.5$
$-\frac{1}{2}$	$\frac{1}{-1/2} = -2$

End behavior :

$x \to \infty$ $\quad$ $x \to -\infty$

$y \to 0$ $\quad$ $y \to 0$

we will relate them to horizontal asymptotes

$x \to 0^+$ (approaches 0 from the right)

$y \to \infty$

$x \to 0^-$

$y \to -\infty$

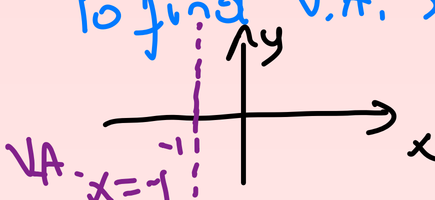
we will this to vertical asymptotes

Begin by factoring the numerator and the denominator. If both the denominator and the numerator have a common factor that reduces, then we say that the rational function has a **hole** at the corresponding x of that factor.

$f(x) = \frac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x+1)}$ we say there is a hole at $x=1$.

Vertical Asymptotes - A rational function has vertical asymptotes where the denominator is zero (after any cancellation of common factors from the denominator and numerator)

$f(x) = \frac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x+1)} = \frac{x+2}{x+1}$ To find V.A. set $x+1=0$, $x=-1$



Horizontal asymptotes

Case 1: If the degree of the numerator is less than the degree of the denominator, the the horizontal asymptote is $y = 0$.

$f(x) = \frac{x+1}{x^2+2}$ → degree is 1, → degree 2

H.A. is $y=0$ (the x axis)

$f(x) = \frac{x^3+x+1}{x^5-4}$ $y=0$ is H.A.

Case 2: If the degree of the numerator equals the degree of the denominator, then the horizontal asymptote is $y =$ the ratio of the leading coefficients.

$f(x) = \frac{1 \cdot x^3+1}{2x^3+3}$ HA. is $y = 1/2$

Case 3: Lastly, if the degree of the numerator is higher than the degree of the denominator, then the rational function has no horizontal asymptotes.

$f(x) = \frac{x^5+4}{x^2+3}$ no H.A

Find the asymptotes and holes(if any) of the rational function
$$f(x) = \frac{(x^2-1)(2x+3)}{(x+1)(x^2+5x+6)} = \frac{(x-1)(x+1)\,(2x+3)}{(x+1)\,(x+2)(x+3)}$$

hole: $f(x) = \frac{(x-1)\cancel{(x+1)}(2x+3)}{\cancel{(x+1)}(x+2)(x+3)}$

at $x=-1$ (since $(x+1)$ factor cancels)

y coordinate of hole is at $f(-1)$

$$f(x) = \frac{(x-1)(2x+3)}{(x+2)(x+3)}$$

$$f(-1) = \frac{(-1-1)(-2+3)}{(-1+2)(-1+3)} = \frac{(-2)}{2} = -1$$

hole @ $(-1,-1)$

VA: $f(x) = \frac{(x-1)(2x+3)}{(x+2)(x+3)}$

where denominator is zero

so VA @ $x=-2$, $x=-3$

HA $f(x) = \frac{(x-1)(2x+3)}{(x+2)(x+3)}$

$$\boxed{y = \frac{2}{1}}$$

degree of top = degree of bottom