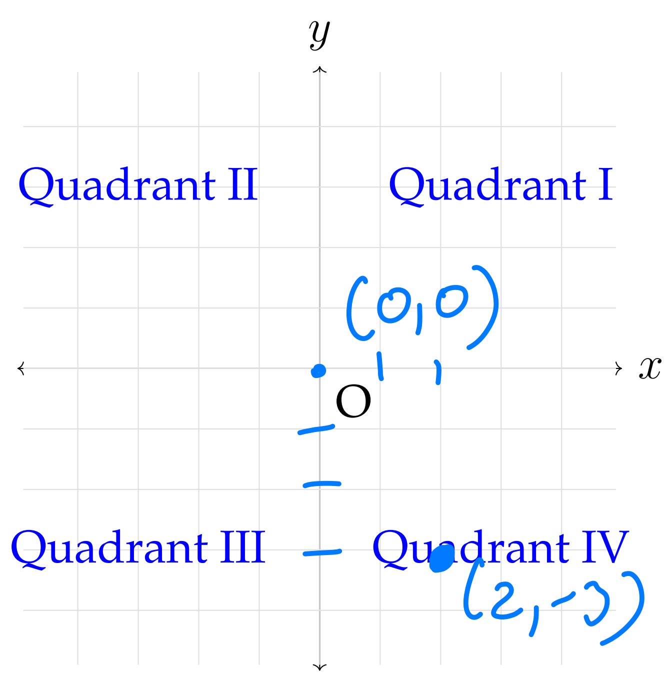
Day 4 Notes
2.1 The coordinate system and Circles

1. Introduction to the Coordinate System

The rectangular coordinate system (also called the Cartesian plane) consists of two perpendicular number lines:

- The horizontal line is called the x axis
- The vertical line is called the y axis
- The point where these lines intersect is called the origin

(2,-3)



Quadrants:

- Quadrant I: Both x and y are positive
- Quadrant II: x is negative and y is positive
- Quadrant III: Both x and y are negative
- Quadrant IV: x is positive and y is negative

Plotting Points: Any point in the plane can be written as an ordered pair (x, y) where:

- x represents the number of units in the x direction
- y represents the -//- in the y direction

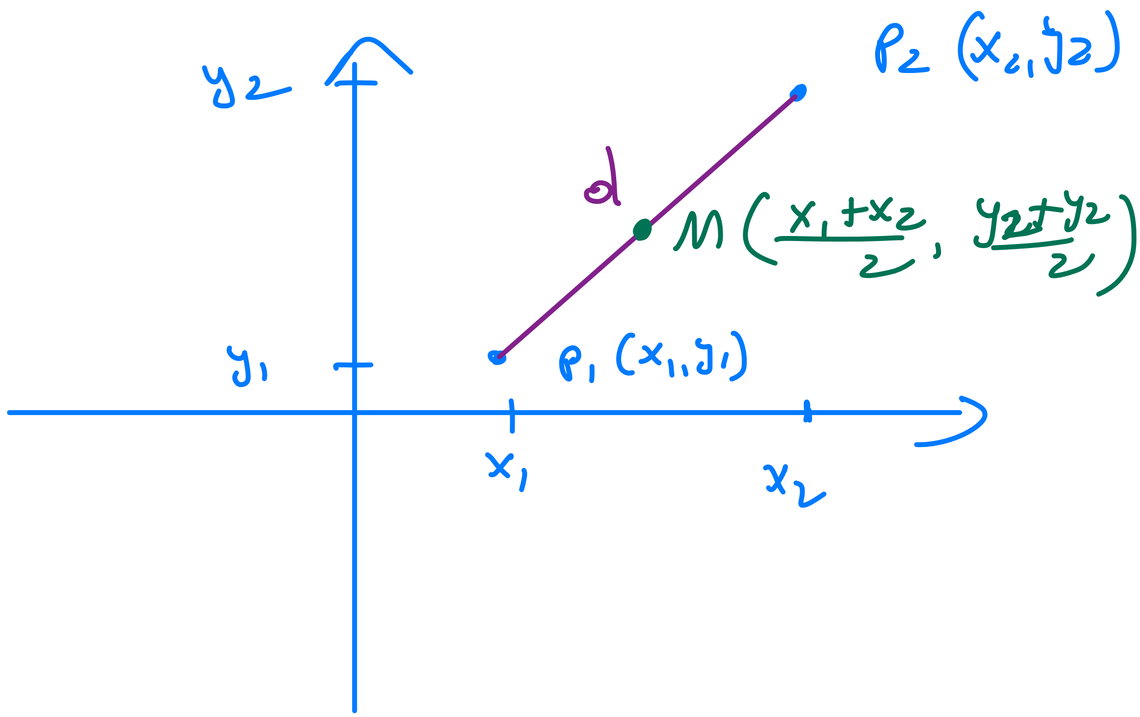
2. Distance and Midpoint Formulas

Distance Formula: The distance d between two points (x_1, y_1) and (x_2, y_2) is:

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Midpoint Formula: The midpoint $M(x, y)$ of a line segment with endpoints (x_1, y_1) and (x_2, y_2) is:

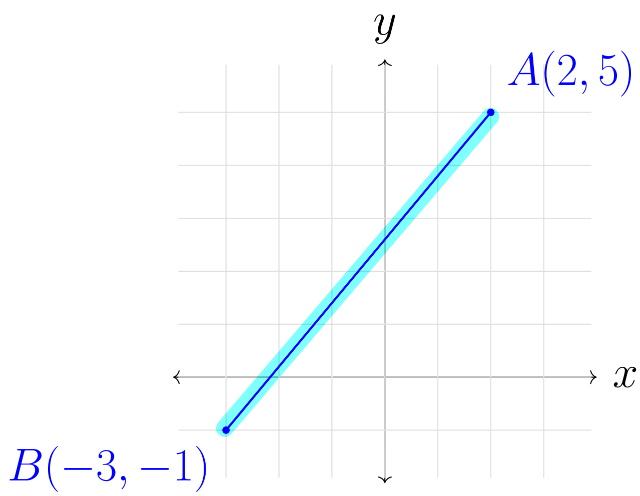
$$\boxed{M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$



3. Examples

$x_1\ y_1$ $\qquad$ $x_2\ y_2$

Example 1: Find the distance between points $A(2,5)$ and $B(-3,-1)$



Solution:

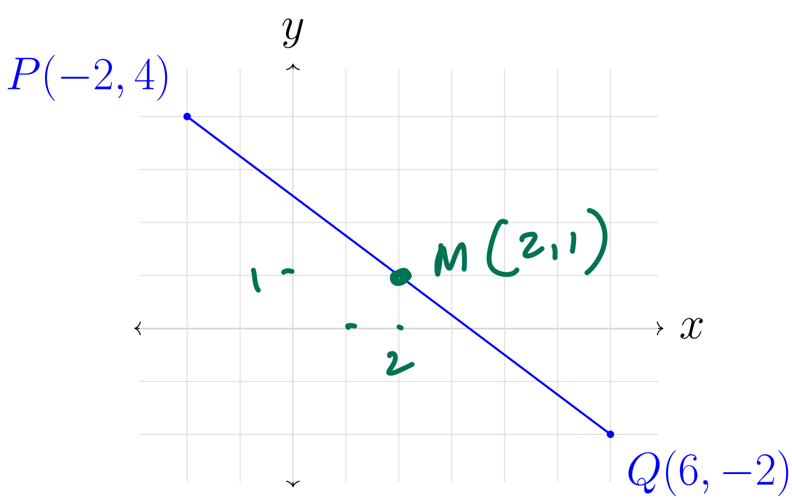
$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(-3-2)^2+(-1-5)^2}$$

$$= \sqrt{(-5)^2+(-6)^2}$$

$$= \sqrt{25+36}$$

$$d = \sqrt{61}$$

Example 2: Find the midpoint of the line segment with endpoints $P(-2,4)$ and $Q(6,-2)$

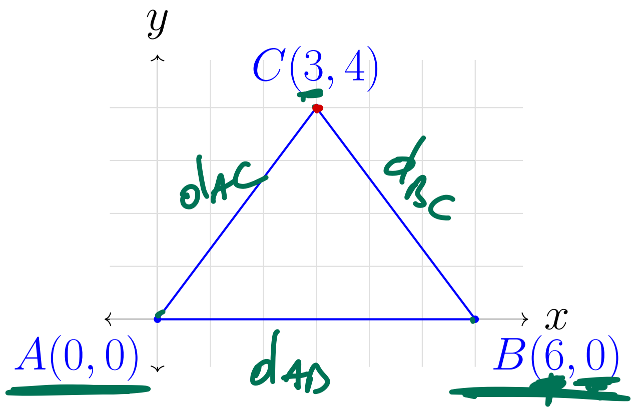


Solution:

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \rightarrow M\left(\frac{-2+6}{2}, \frac{4+(-2)}{2}\right)$$

$$M(2,1)$$

Example 3: Determine if triangle ABC with vertices $A(0,0)$, $B(6,0)$, and $C(3,4)$ is a right triangle.



not a right triangle.

Solution:

$$d_{AB} = \sqrt{(6-0)^2+(0-0)^2} = \sqrt{36} = 6$$

$$d_{BC} = \sqrt{(6-3)^2+(0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

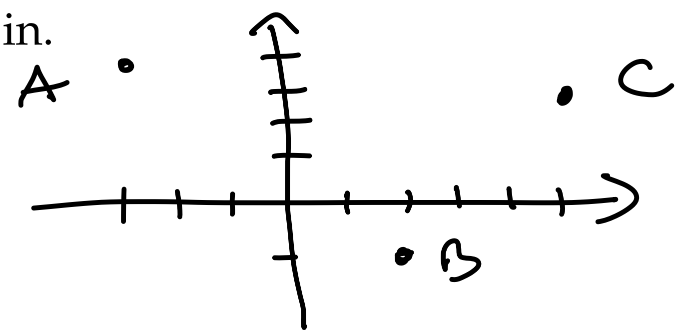
$$d_{AC} = \sqrt{(0-3)^2+(0-4)^2} = \sqrt{9+16} = 5$$

$$\cancel{5^2+5^2=6^2 \qquad 5^2+6^2=5^2}$$

Practice Problems:

II IV I

1. Plot the points $A(-3,4)$, $B(2,-1)$, and $C(5,3)$. Label which quadrant each point is in.



2. Find the distance between points $(1,7)$ and $(4,-2)$ and the midpoint between them.

$$d = \sqrt{(4-1)^2+(-2-7)^2} = \sqrt{3^2+(-9)^2} = \sqrt{9+81} = \sqrt{90}$$

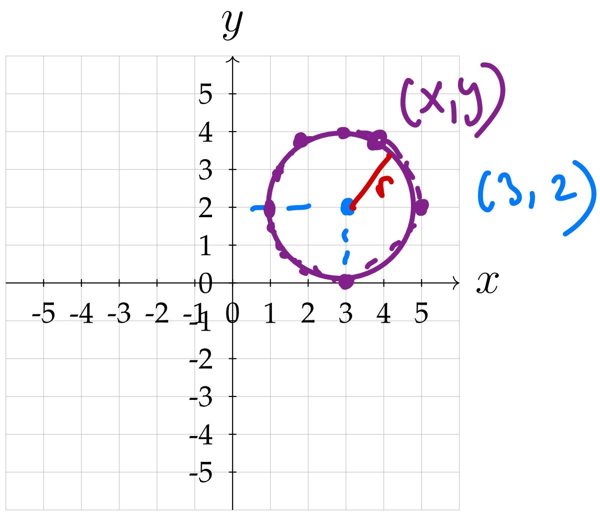
$$M\left(\frac{1+4}{2}, \frac{7-2}{2}\right) \rightarrow M\left(\frac{5}{2}, \frac{5}{2}\right)$$

1. Circles

Next, we will start graphing specific categories of equations such as circles, ellipses, hyperbolas and parabola. We begin with circles.

same distance

Definition: A circle is the set of all points in a plane that are equidistant from a fixed point called center. The fixed distance from any point on the circle to the center is called the radius.



r - radius

r = 2

center : (h, k)

k → y coord of center

h → x coordinate of center

Standard form of an equation of a circle

Given a circle centered at (h, k) with radius r, the standard form is

$$\boxed{(x-h)^2+(y-k)^2=r^2}$$

standard formula for a circle

Example 1

1. $(x-4)^2 + (y+3)^2 = 25$

$(x-4)^2 + (y-(-3))^2 = 5^2$

center: $(4,-3)$
radius: $r=5$

2. $x^2 + (y-1/2)^2 = 12$

$(x-0)^2 + (y-1/2)^2 = (\sqrt{12})^2$

center: $(0, 1/2)$
radius: $r = \sqrt{12} = \sqrt{4 \cdot 3}$
$r = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

3. $x^2 + y^2 = 7$

$(x-0)^2 + (y-0)^2 = (\sqrt{7})^2$

$(0,0)$
$r = \sqrt{7}$

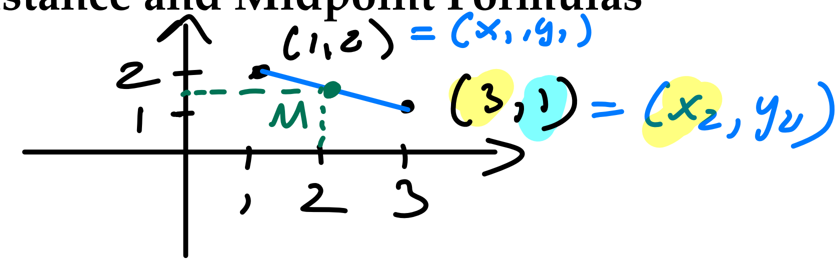
Example 2 Write the standard form of an equation of a circle with center $(-4, 6)$ and radius 2.

$(x-h)^2 + (y-k)^2 = r^2$

$(x-(-4))^2 + (y-6)^2 = 2^2$

$$\boxed{(x+4)^2 + (y-6)^2 = 4}$$

2. Recall: Distance and Midpoint Formulas



Distance Formula: The distance d between two points (x_1, y_1) and (x_2, y_2) is:

$d = \sqrt{(3-1)^2 + (1-2)^2} = \sqrt{4+1} = \sqrt{5}$

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

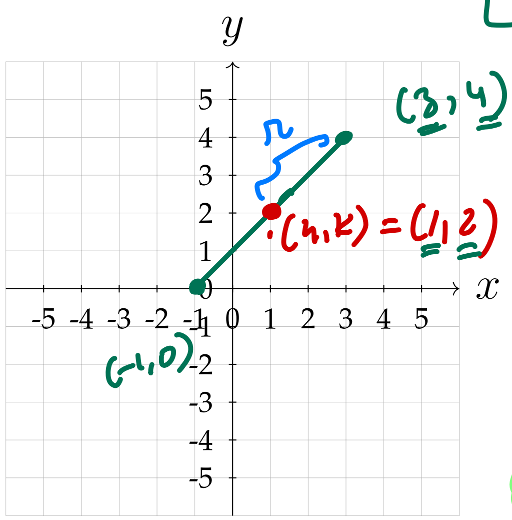
Midpoint Formula: The midpoint $M(x, y)$ of a line segment with endpoints (x_1, y_1) and (x_2, y_2) is:

$$\boxed{M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

$M\left(\frac{1+3}{2}, \frac{2+1}{2}\right) = \left(\frac{4}{2}, \frac{3}{2}\right) = (2, 1.5)$

Example 3 Write the standard form of an equation of a circle with the endpoints of the diameter as $(-1, 0)$ and $(3, 4)$. Graph the circle.

$\hookrightarrow$ twice the radius



$(x-h)^2+(y-k)^2=r^2$

$(h,k) = M\left(\frac{-1+3}{2}, \frac{0+4}{2}\right) = (1,2)$

$(x-1)^2+(y-2)^2=(\sqrt{8})^2$

$r=\sqrt{(3-1)^2+(4-2)^2} = \sqrt{4+4} = \sqrt{8} = \sqrt{4\cdot 2}$

$r = 2\sqrt{2}$

$(x-1)^2+(y-2)^2=8$

General form of an Equation of a circle

$(h,k) = ?$
$r = ?$

An equation of a circle written in the form $x^2 + y^2 + Ax + By + C = 0$ is called the general form of an equation of a circle.

Example 4 Write the equation of the circle in standard form by **completing the square.**

$\boxed{x^2 + y^2 + 10x - 6y + 25 = 0}$ general form $\rightarrow$ $\boxed{(x-h)^2+(y-k)^2=r^2}$ standard form

1) $\underbrace{x^2+10x}_{\text{binomial in } x} + \underbrace{y^2-6y}_{\text{binomial in } y} = -25$

2) $x^2+10x+25 + y^2-6y+9 = -25+25+9$

take half of coeff of x, square it and add to both sides

$\left(\frac{10}{2}\right)^2 = 5^2 = 25$

$\left(-\frac{6}{2}\right)^2 = (-3)^2 = 9$

3) $(x+5)^2+(y-3)^2=9$

Steps for completing the square

1. Rewrite the general equation by grouping the x terms together and y terms together
2. Complete the square for binomial in x and for binomial in y.