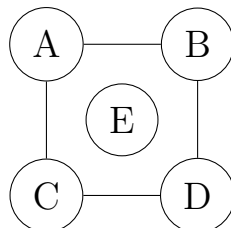


MTH 140: Exam 1

Name: Key

1. (12 points) Use the following graph to answer the questions:

1 (a) The degree of vertex A is 2.1 (b) The degree of vertex E is 0.2 (c) Is the graph connected? NO

2 (d) Give an example of two adjacent vertices.

A and B

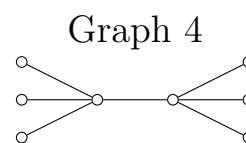
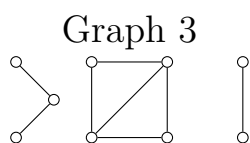
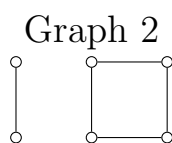
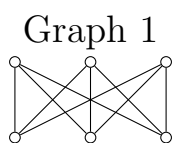
2 (e) Give an example of two adjacent edges.

AB and AC

2 (f) Give an example of two different paths from A to D.

ABD and ACD2 (g) Give an example of ~~two~~ ^{one} different circuits.ABDC

2. (5 points) Use the figure below to answer the question.



Which graphs are connected? Circle all that apply.

a. Graph 1

b. Graph 2

c. Graph 3

d. Graph 4

e. None of the above

3. (3 points) The degrees of all vertices of a graph add up to 16. How many edges does the graph have?

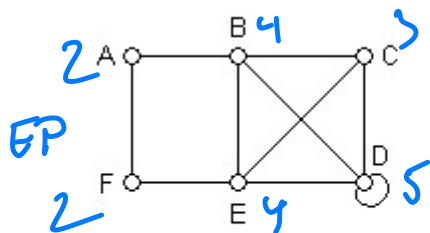
a. 6 edges

b. 16 edges

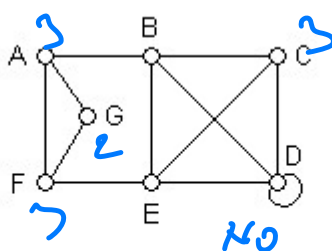
c. 8 edges

d. 4 edges

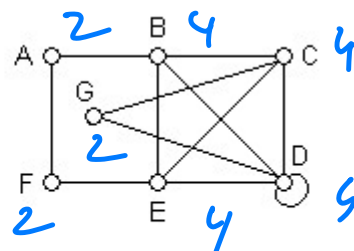
4. (5 points) Use the figure below to answer the question. Which of the graphs has an Euler path but no Euler circuits?



Graph 1



Graph 2



Graph 3

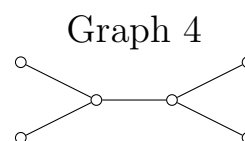
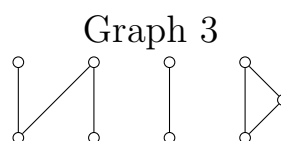
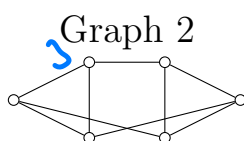
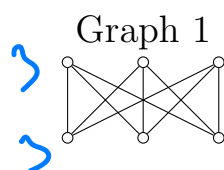
a. Graph 1

b. Graph 2

c. Graph 3

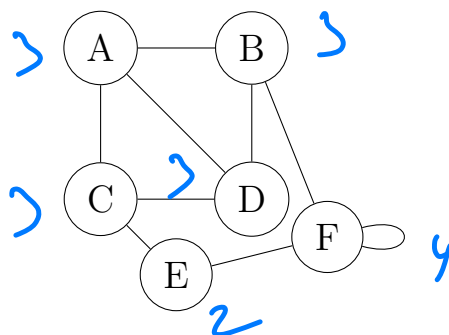
d. None of the above

5. (5 points) Which graph has an Euler circuit?



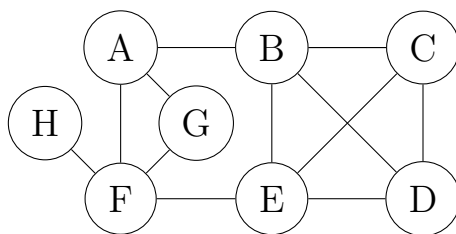
- a. Graph 1
- b. Graph 2
- c. Graph 3
- d. Graph 4
- e. None of the above

6. (5 points) Given the graph, which of the following statements is true?



- a. The graph has an Euler path.
- b. The graph has an Euler circuit.
- c. The graph has both an Euler circuit and an Euler path.
- d. The graph has neither an Euler circuit nor an Euler path.

7. (7 points) Consider the graph below.



2 (a) Is the graph a network?

yes

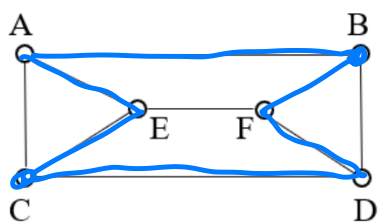
2 (b) Does the graph have any bridges? which one?

yes HF

3 (c) Does the graph have a Hamilton circuit. If, yes list one Hamiltonian circuit.

NO

8. (6 points) Does the graph have a Hamiltonian circuit? If, yes write it down.



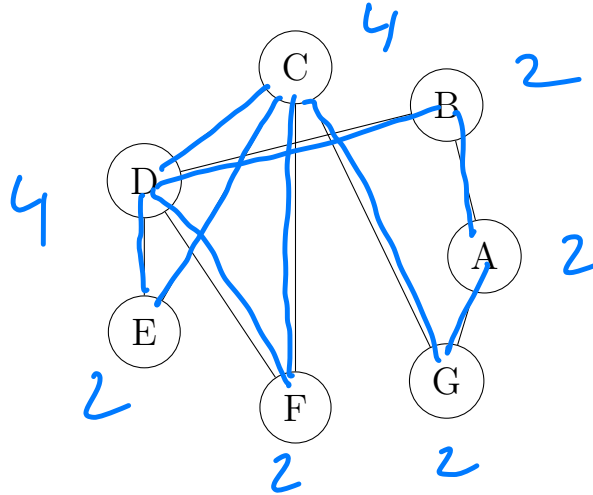
BFDC EAB

9. (4 points) How many edges are in the complete graph K_{10} ?

$$(10-1)! = 9!$$

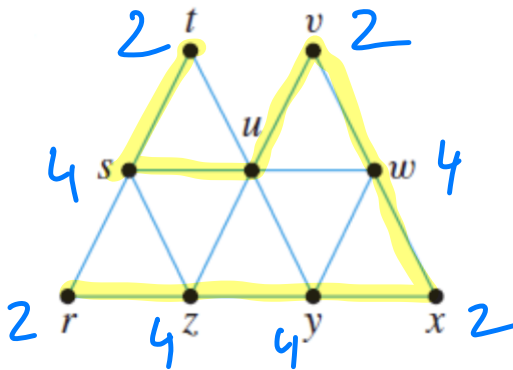
$$K_8 \text{ has } (8-1)! = 7!$$

10. (6 points) After a storm, the city crew inspects for trees or brush blocking the road. Find an efficient route for the neighborhood below by finding an Euler circuit. *write down*



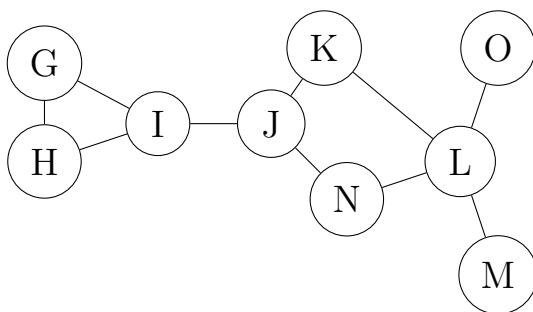
CDEC FD BAGC

11. (5 points) Considering the graph, which of the following is true?



- a. The graph has both an Euler circuit and a Hamiltonian circuit.
 b. The graph has an Euler circuit, but not a Hamiltonian circuit
 c. The graph has a Hamiltonian circuit, but not an Euler circuit.
 d. The graph has neither an Euler circuit nor a Hamiltonian circuit.

12. (10 points) Consider the following graph.



2 (a) How many vertices does the graph have?

add)

2 5) 9 v

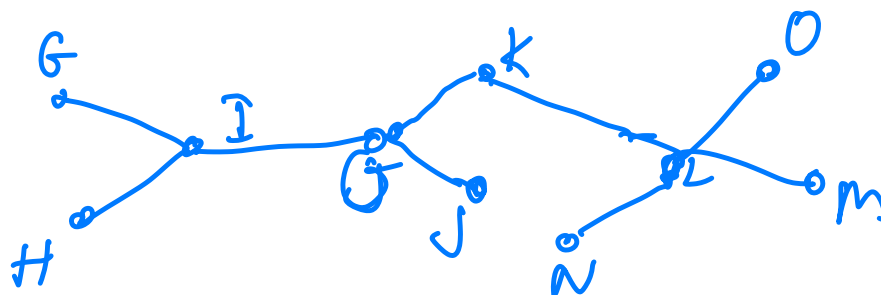
2 5) (b) How many edges should a tree corresponding to this graph have?

1 8 edges

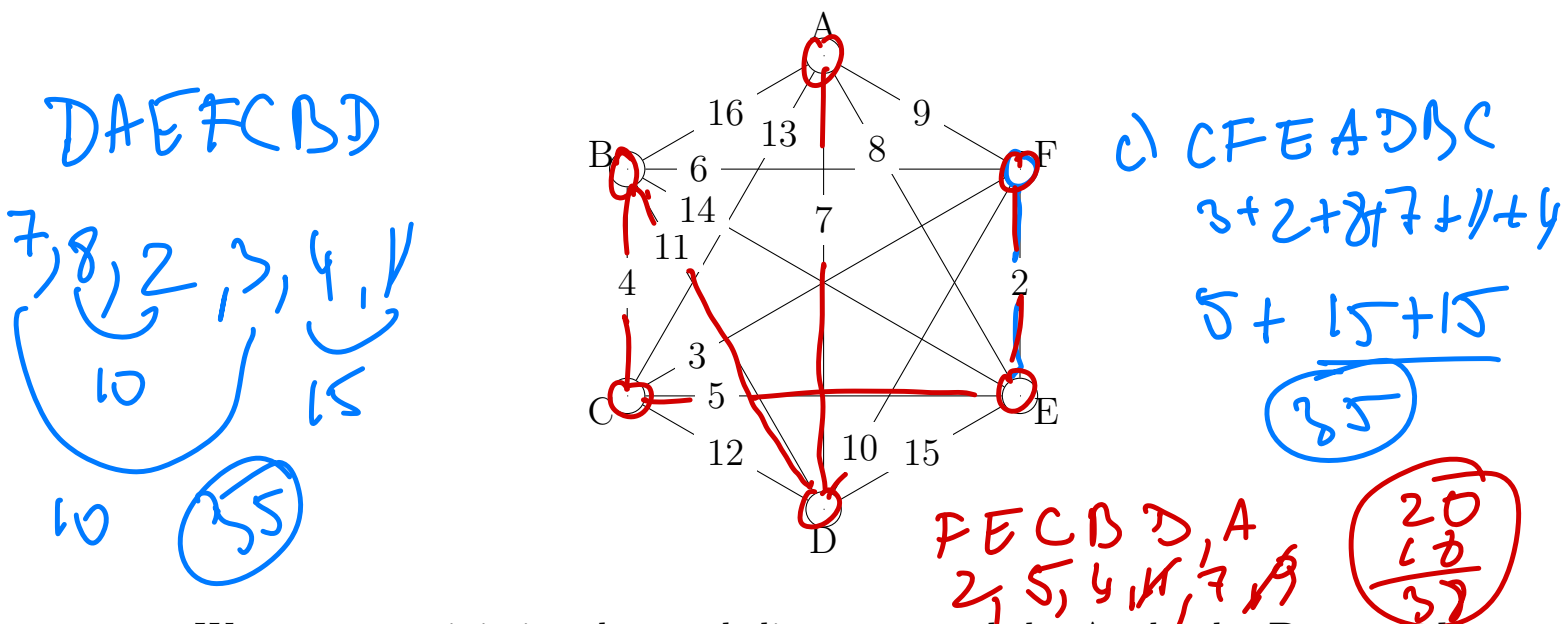
2 4) (c) How many spanning trees does this graph have?

$3 \cdot 4 = 12$

3 4) (d) Draw one spanning tree corresponding to this graph.



13. (12 points) A delivery truck must deliver packages to 6 different store locations (A, B, C, D, E, and F). The trip must start and end at A. The graph below shows the distances (in miles) between locations:

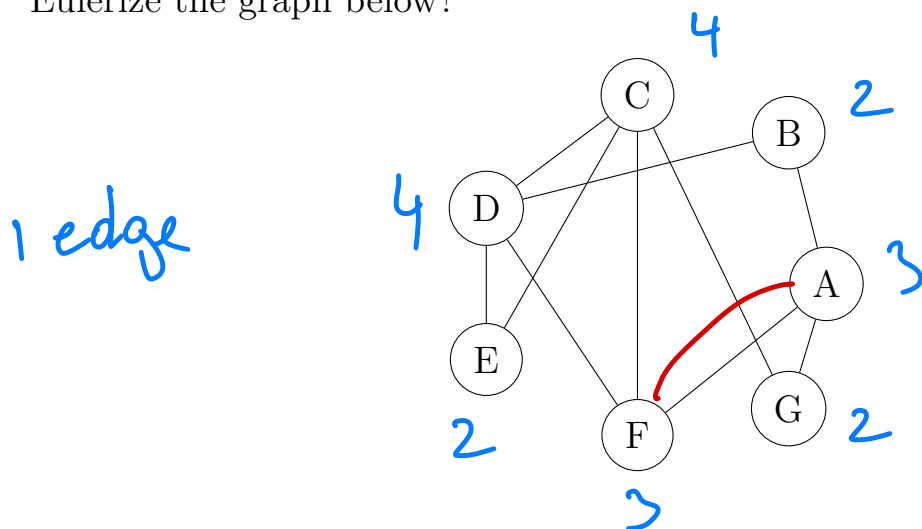


We want to minimize the total distance traveled. Apply the **Repeated Nearest Neighbour Algorithm** at the following vertices.

- (a) Circuit starting at A: A E F C B D A
 $8 + 2 + 3 + 4 + 11 + 7 = 35$
- (b) ~~Circuit starting at B~~
~~B C F E A D B: 4 + 3 + 2 + 8 + 7 + 11 = 37~~
- (c) Circuit starting at C
~~= 7 + 10 + 18 = 35~~
- (d) ~~Circuit starting at D~~ 35
- (e) ~~Circuit starting at E~~ E F C B D E
 $2, 3, 4, 11, 15$ 35
- (f) Circuit starting at F

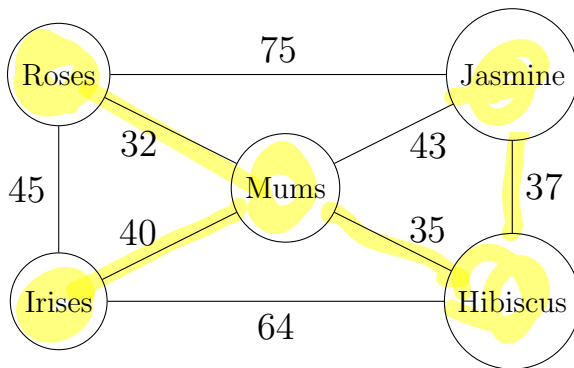
- (g) What is the solution (optimal circuit starting and ending at A)? What is the minimum distance the driver must cover?

14. (5 points) What is the least number of edges that must be duplicated to Eulerize the graph below?



15. (10 points) Kenzie wants to install an irrigation system to water her five flower gardens. The distance in feet between her gardens are shown in the graph below.

Use Kruskal's algorithm to find the minimal spanning tree and its ~~cost~~.



- 1) R M
- 2) M H
- 3) H J
- 4) I M

cost

$$32 + 40 + 35 + 37$$

$$144 \times 45$$

\$6480

Question	Points	Score
1	12	
2	5	
3	3	
4	5	
5	5	
6	5	
7	7	
8	6	
9	4	
10	6	
11	5	
12	10	
13	12	
14	5	
15	10	
Total:	100	