MTH 142: Practice Exam 3 Key

1. The library is to be given 5 books as a gift. The books will be selected from a list of 21 titles. If each book selected must have a different title, how many possible selections are there?
   - a. 4.258×10^{17}
   - b. 5.109×10^{19}
   - **c.** $20,349$
   - d. $2,441,880$

2. Of the $2,598,960$ different five-card poker hands possible from a deck of 52 playing cards, how many would contain 2 black cards and 3 red cards?
   - **a.** $845,000$
   - b. $1,267,500$
   - c. $422,500$
   - d. $1,690,000$

3. There are 9 members on a board of directors. If they must form a subcommittee of 6 members, how many different subcommittees are possible?
   - a. $531,441$
   - b. 720
   - c. $60,480$
   - **d.** 84

4. Assume that a researcher randomly selects 14 newborn babies and counts the number of girls selected, X. Use the following probability table to find the following probabilities.

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$P(X = x)$	.00	.001	.006	.022	.061	.122	.183	.183	.171	.134	.088	.022	.006	.001	.0

   (a) Find probability of selecting more than 10 girls.
   - a. 0.061
   - b. 0.903
   - c. 0.212
   - **d.** 0.029

   (b) Find probability of selecting less than 3 girls.

5. The random variable x represents the number of tests that a patient entering a hospital will have along with the corresponding probabilities. Find the probability that a patient has fewer than two tests.

x	P(x)
0	3/17
1	5/17
2	4/17
3	3/17
4	2/17

a. $\frac{6}{17}$
b. $\frac{5}{17}$
c. $\frac{8}{17}$
d. $\frac{3}{17}$

6. Assume that a researcher randomly selects 14 newborn babies and counts the number of girls selected, X. Use the probability table from Question 4 to find the probability of selecting fewer than 6 girls.

a. 0.183
b. 0.817
c. 0.212
d. 0.395

7. Find the mean of the following probability distribution:

x	P(x)
2	0.33
3	0.24
4	0.43

a. 1.60
b. 2.10
c. 2.60
d. 3.10

8. Find the standard deviation of the following probability distribution:

x	P(x)
0	0.20
2	0.05
4	0.35
6	0.25
8	0.15

a. 2.6
b. 4.7
c. 5.4
d. 3.9

9. At a raffle, 10,000 tickets are sold at \$5 each for three prizes valued at \$4,800, \$1,200, and \$400. What is the expected value of one ticket?

   a. -\$0.64

   b. \$0.64

   c. \$4.36

   d. -\$4.36

10. In a study, 40% of adults questioned reported that their health was excellent. A researcher wishes to study the health of people living close to a nuclear power plant. Among 10 adults randomly selected from this area, only 3 reported that their health was excellent. Find the probability that when 10 adults are randomly selected, 3 or fewer are in excellent health.

    a. 0.2687

    b. 0.1673

    c. 0.2150

    d. 0.3823

11. The probability that a tennis set will go to a tiebreaker is 16%. In 420 randomly selected tennis sets, what is the mean and the standard deviation of the number of tiebreakers?

    a. mean: 63; standard deviation: 8.2

    b. mean: 63; standard deviation: 7.51

    c. mean: 67.2**; standard deviation:** 7.51

    d. mean: 67.2; standard deviation: 8.2

12. The chance of an IRS audit for a tax return with over \$125,000 in income is about 3%. A random sample of 60 taxpayers with incomes above \$125,000 is selected. Use this information to answer the following:

    (a) How many taxpayers in the sample are expected to be audited?

    (b) What is the probability that exactly 4 of the selected taxpayers will be audited?

    (c) What is the probability that at most 4 of the selected taxpayers will be audited?

    (d) What is the probability that more than 2 of the selected taxpayers will be audited?

(e) A distribution $X \sim U(2, 15)$. Find $P(3.5 < x < 8)$. Round answer to three decimal places.

a. 0.654
b. 0.346
c. 0.467
d. 0.300

> **Solution:** For uniform distribution: $P(3.5 < x < 8) = \frac{8-3.5}{15-2} = \frac{4.5}{13} = 0.346$

(f) Suppose you arrive into a building and are about to take an elevator to your floor. Once you call the elevator, it will take between 2 and 40 seconds to arrive to you. We will assume that the elevator arrives uniformly between 2 and 40 seconds after you press the button. Find μ.

a. 19
b. 21
c. 11
d. 22

> **Solution:** For uniform distribution, $\mu = \frac{a+b}{2} = \frac{2+40}{2} = 21$

(g) A tire company finds the lifespan for one brand of its tires is normally distributed with a mean of $48,400$ miles and a standard deviation of 5000 miles. If the manufacturer is willing to replace no more than 10% of the tires, what should be the approximate number of miles for a warranty?

a. $55,000$
b. $40,000$
c. $42,000$
d. $57,000$

(h) An airline knows from experience that the distribution of the number of suitcases that get lost each week on a certain route is approximately normal with $\mu = 15.5$ and $\sigma = 3.6$. What is the probability that during a given week the airline will lose more than 20 suitcases?

a. 0.1056
b. 0.3944
c. 0.8944
d. 0.4040

(i) The distribution of cholesterol levels in teenage boys is approximately normal with $\mu = 170$ and $\sigma = 30$.

(a) Find the probability that a randomly selected boy has a cholesterol level between 160 and 190 days.

> **Solution:** $P(160 < X < 190) = 0.3781$

(b) If a sample of 35 medical records are randomly selected, what is the probability that the mean length of the pregnancies is between 160 and 190 days?

> **Solution:** Using CLT: $P(160 < \bar{X} < 190) = 0.9757$

(c) Find the cholesterol level that is at the 70th percentile.

Solution: 185.73

(d) The amount of snowfall falling in a certain mountain range is normally distributed with a mean of 105 inches and a standard deviation of 10 inches. What is the probability that the mean annual snowfall during 25 randomly picked years will exceed 107.8 inches?

a. 0.0808

b. 0.4192

c. 0.0026

d. 0.5808

Solution: Using CLT with $\sigma_{\bar{x}} = \frac{10}{\sqrt{25}}$ you should get 0.0808

(e) Use the given information to find the mean and standard deviation for the sampling distribution of $\bar{x}$. The mean and the standard deviation of the sampled population are, $\mu = 77.4$ and $\sigma = 4.0$. Sample size is $n = 225$

a. $\mu = 0.3$; $\sigma = 77.4$

b. $\mu = 77.4$**;** $\sigma = 0.3$

c. $\mu = 20.6$; $\sigma = 0.8$

d. $\mu = 4.0$; $\sigma = 0.3$

Solution: For sampling distribution: $\mu_{\bar{x}} = \mu = 77.4$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4.0}{\sqrt{225}} = 0.3$

(f) For samples of the specified size from the population described, find the mean and standard deviation of the sample mean $\bar{x}$. The National Weather Service keeps records of rainfall in valleys. Records indicate that in a certain valley, the annual rainfall has a mean of 70 inches and a standard deviation of 12 inches. For samples of size 36, determine the mean and standard deviation of $\bar{x}$.

a. $\mu = 70$**;** $\sigma = 2$

b. $\mu = 12$; $\sigma = 70$

c. $\mu = 70$; $\sigma = 12$

d. $\mu = 2$; $\sigma = 70$

Solution: For sampling distribution: $\mu_{\bar{x}} = \mu = 70$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{36}} = 2$

(g) When 345 college students are randomly selected and surveyed, it is found that 110 own a car. Use this data to find a 99% confidence interval for the true proportion of all college students who own a car.

a. $0.254 < p < 0.383$

b. $0.270 < p < 0.368$

c. $0.260 < p < 0.377$

d. $0.278 < p < 0.360$

Solution: $\hat{p} = \frac{110}{345} = 0.319$ For 99% CI: $z^* = 2.576$ CI $= \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.319 \pm 2.576\sqrt{\frac{0.319(0.681)}{345}} = 0.319 \pm 0.065 = (0.254, 0.383)$

(h) A sample of 15 randomly selected students from a certain college has a grade point average of 2.86 with a standard deviation of 0.78. Assuming the population has a normal distribution, use the sample data to construct a 90% confidence interval for the population mean, μ, of GPA's for all students at this college.

a. $(2.51, 3.21)$
b. $(2.37, 3.56)$
c. $(2.41, 3.42)$
d. $(2.28, 3.66)$

Solution: For 90% CI with df $= 14$: $t^* = 1.761$ CI $= \bar{x} \pm t^* \frac{s}{\sqrt{n}} = 2.86 \pm 1.761 \frac{0.78}{\sqrt{15}} = 2.86 \pm 0.35$ $= (2.51, 3.21)$

(i) A botanist wishes to estimate the average number of seeds for a certain fruit. She randomly selects 48 pieces of this fruit, and counts the number of seeds in each. The sample produced a mean of 22.7 seeds with a standard deviation of 12.6 seeds. Assume that the number of seeds per fruit is normally distributed.

(a) Use the sample data to find a 95% confidence interval for the mean number of seeds per fruit.

Solution: For 95% CI with df $= 47$: $t^* = 2.011$ CI $= 22.7 \pm 2.011 \frac{12.6}{\sqrt{48}} = 22.7 \pm 3.659$ $= (19.041, 26.359)$

(b) What is the margin of error for this estimate?

Solution: ME $= 3.659$

(c) Suppose we wish to estimate the mean number of seeds per fruit with 95% confidence and a margin of error of 2 seeds. What would be the minimum sample size required?

Solution: $n = \left(\frac{z^* \sigma}{E}\right)^2 = \left(\frac{1.96(12.6)}{2}\right)^2 = 152.47$ Rounding up, $n = 153$

(d) Find the necessary sample size to estimate a mean to within \$123 with 95% confidence if the standard deviation is \$546.

a. 67

b. 76

c. 4

d. 2

Solution: Using the formula: $n = \left(\frac{z^* \sigma}{E}\right)^2$ For 95% confidence, $z^* = 1.96$ $n = \left(\frac{1.96(546)}{123}\right)^2 =$ 75.698 Rounding up, $n = 76$