Day 25 Notes
Inverse Matrices and Matrix Equations

1. Inverse Matrices

In this section we will learn how to solve systems of linear equations by using inverse matrices.

A square matrix A has an inverse if there exists another matrix B such that $AB = I$ and $BA = I$ where I in an identity matrix.

Example:
Verify that B is the inverse of A, where

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

2. Finding the inverse of a 2x2 Matrix

The following rule provides a simple way for finding the inverse of a 2x2 matrix, when it exists.

INVERSE OF A 2 × 2 MATRIX

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

If $ad - bc = 0$, then A has no inverse.

Example:

Find the inverse of $A = \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}$

3. Finding inverses for higher order matrices

If A is an n x n matrix, we first construct the n x 2n matrix that has the entries of A on the left and of the identity matrix I_n on the right:

$$\left[\begin{array}{cccc:cccc} a_{11} & a_{12} & \cdots & a_{1n} & 1 & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & 0 & 0 & \cdots & 1 \end{array}\right]$$

We then use the elementary row operations on this new large matrix to change the left side into the identity matrix. (This means that we are changing the large matrix to reduced row-echelon form.) Then the inverse of matrix A, denoted A^{-1} is the matrix on the right hand side.

Example: Find the inverse of A

$$A = \begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix}$$

If we encounter a row of zeros on the left when trying to find an inverse, then the original matrix does not have an inverse

4. Solving systems of equations by solving Matrix equations

The system

$$\begin{cases} x - 2y - 4z = 7 \\ 2x - 3y - 6z = 5 \\ -3x + 6y + 15z = 0 \end{cases}$$

is equivalent to the matrix equation

$$\underbrace{\begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 7 \\ 5 \\ 0 \end{bmatrix}}_{B}$$