Day 9 Notes
Graphs of Functions

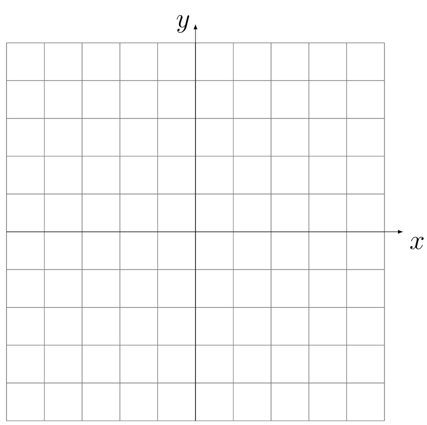
1. Graphs of Functions: Introduction

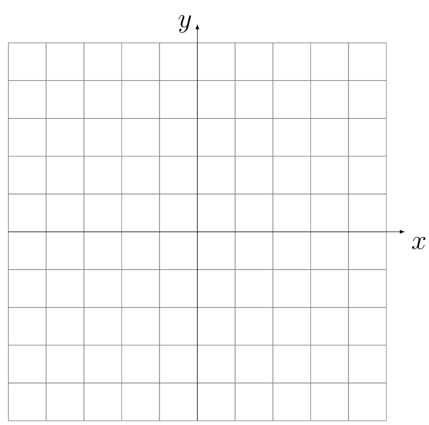
The most important way to visualize a function is through its graph. In this section we investigate in more detail the concept of graphing functions.

To graph of a function f, we plot the points (x, y) in a coordinate plane where the x coordinate represents an input and the y coordinate is the corresponding output of the function, $y = f(x)$.

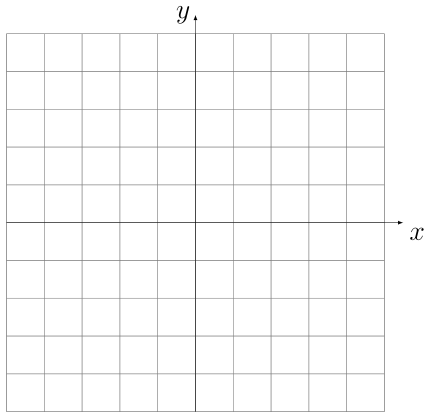
Examples of some functions and their graphs.

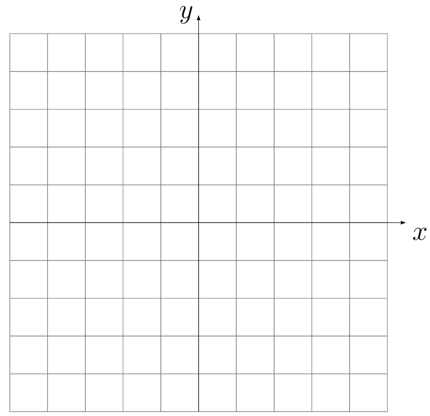
Linear Functions



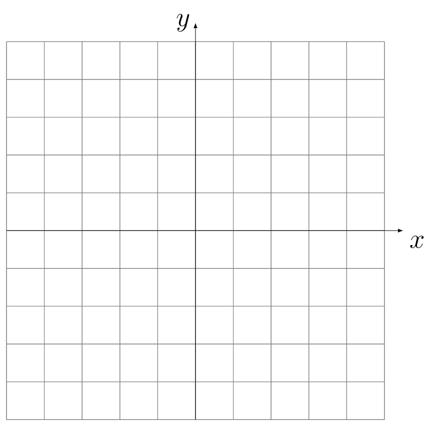


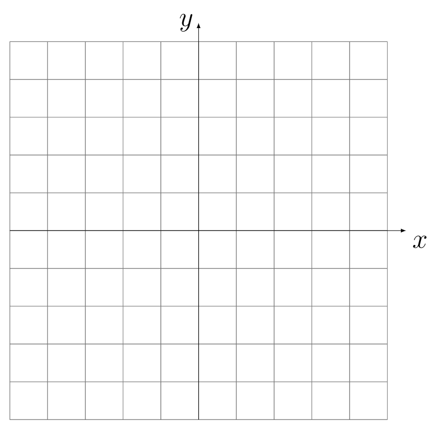
Power Functions - Positive Exponents



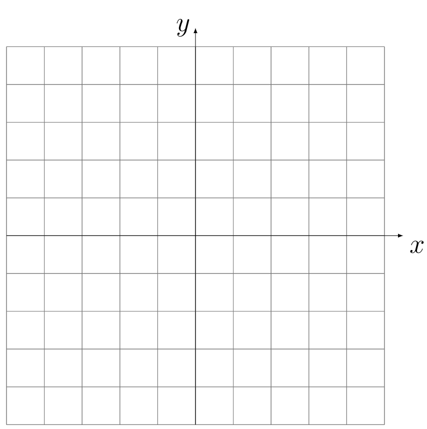


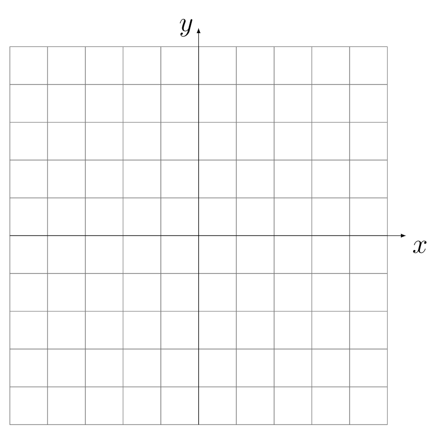
Power Functions - Negative Exponents





Root Functions

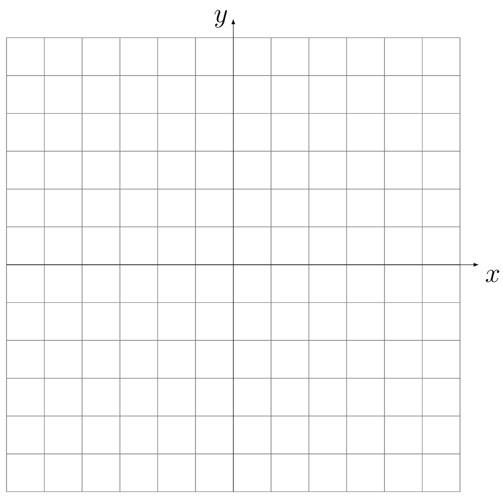




2. Graphing Piecewise defined functions

Graph the function

$$f(x) = \begin{cases} x^2 + 2x & x \leq -1 \\ x & -1 < x \leq 1 \\ -1 & 1 < x \end{cases}$$

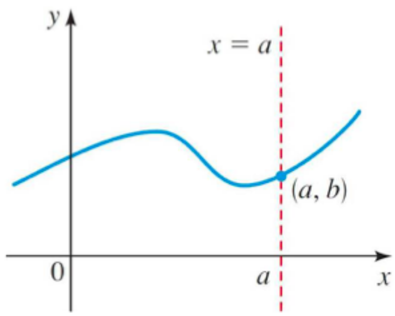


Steps to graph a piecewise function:

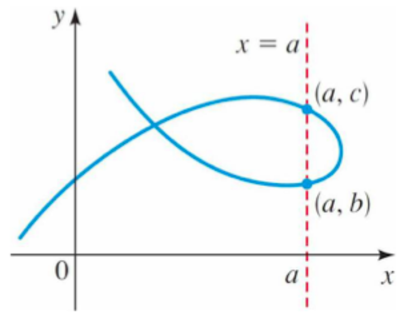
1. Graph each piece on its given domain
2. Use solid dots (•) for included endpoints
3. Use open dots (○) for excluded endpoints
4. Check that the pieces connect as specified

3. Function or not a function?

Given a graph, one can decide if the graph represents a function if the graph passes the **vertical line test**: A curve in the xy-plane represents a function if and only if no vertical intersects the curve more than once.



Graph of a function



Not a graph of a function

Vertical Line Test

Given the equation of the relationship between x and y, one can decide if y represents a function of x by solving for y and seeing if each x value has exactly one y value assigned to it.

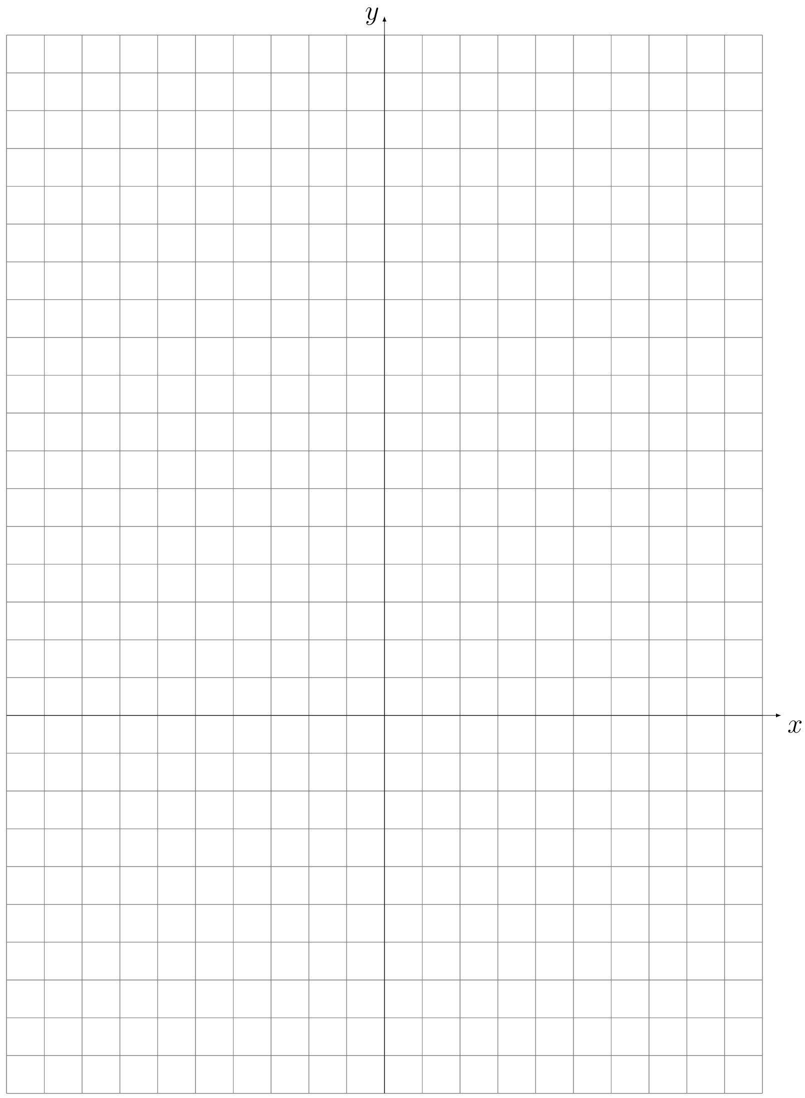
Examples: Does the equation define y as a function of x?

1. $y–x^2 = 2$

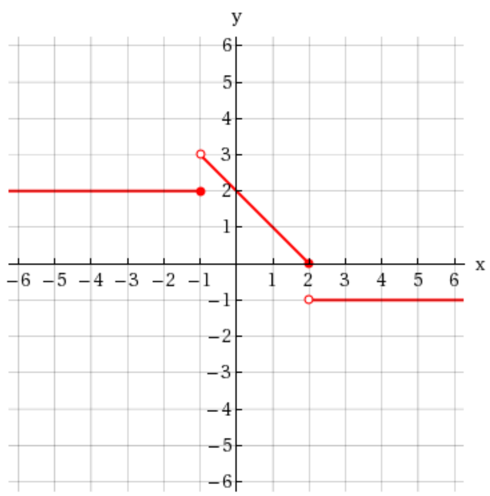
2. $x^2 + y^2 = 4$

Practice: Graph the following function by plotting points.

$$f(x) = \begin{cases} -x & x \leq 0 \\ 9 - x^2 & 0 < x \leq 3 \\ x - 3 & 3 < x \end{cases}$$



Practice: Find a formula for the function and state its domain and range.



Increasing/decreasing/constant function

Let f be a function.

- A function is **increasing** on an interval if as x increases, $f(x)$ increases.

- A function is **decreasing** on an interval if as x increases, $f(x)$ decreases.

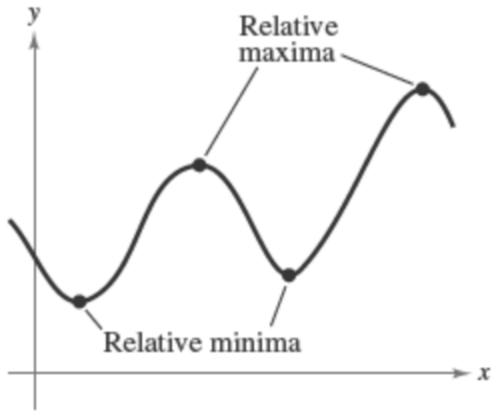
- A function is **constant** on an interval if $f(x)$ remains the same as x changes.

Example 1. Graph the function $f(x) = x^3 + 3x^2 - 1$. Then use the graph to describe the increasing and decreasing behavior of the function.

Relative (local) maxima/minima

For a function f:

- A point is a **local maximum** if $f(x)$ is greater at that point than at nearby points.
- A point is a **local minimum** if $f(x)$ is less at that point than at nearby points



Even/odd functions

- A function is **even** if its graph is symmetric about the y-axis

$$f(-x) = f(x) \text{ for all } x \text{ in the domain}$$

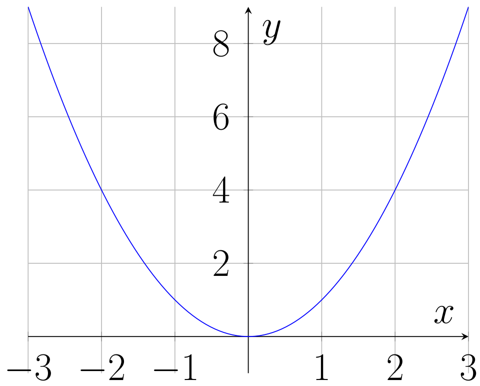
- A function is **odd** if its graph is symmetric about the origin

$$f(-x) = -f(x) \text{ for all } x \text{ in the domain}$$

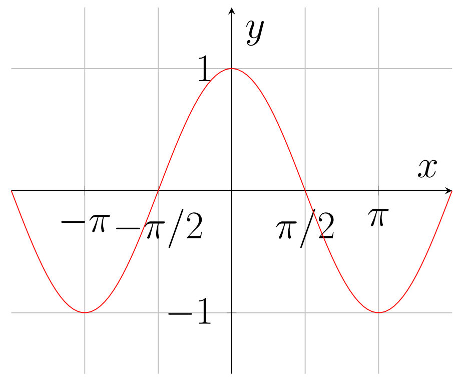
Example 2. Determine whether the function is even, odd or neither.

1. $f(x) = x^3 + 4x$
2. $g(x) = 3x - x^2$
3. $f(t) = 5t^{2/3}$

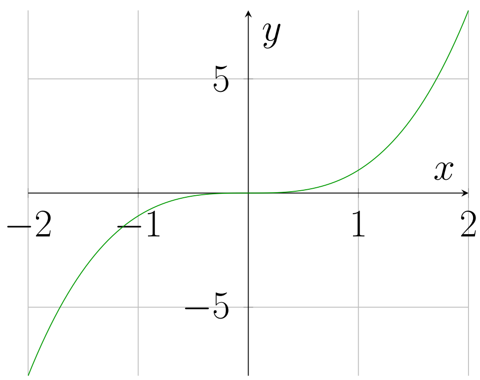
Function: $f_1(x) = x^2$



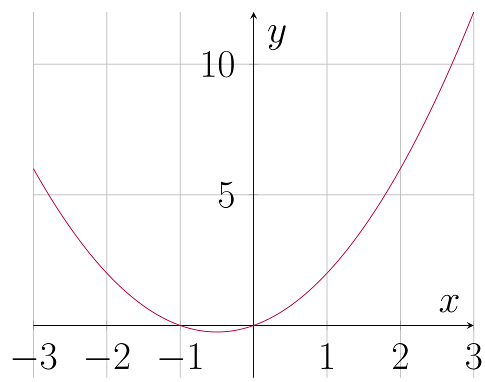
Function: $f_2(x) = \cos(x)$



Function: $f_3(x) = x^3$



$f_4(x) = x^2 + x$



Properties

- For even functions (f_1 and f_2), $f(x) = f(-x)$ for all x in the domain
- For the odd function (f_3), $f(-x) = -f(x)$ for all x in the domain
- For f_4, neither property holds: $f_4(-x) \neq f_4(x)$ and $f_4(-x) \neq -f_4(x)$