p and q : $p \wedge q$; p or q : $p \vee q$

Conditionals and Equivalence guided notes

Conditional: The statement "If p, then q." Symbolized by $p \rightarrow q$ is called a conditional statement. This is also called an implication.

p is called the antecedent (condition)

q is called the consequent (consequence)

Example: If I get my work done by 4 pm, then we will go for a walk.

"I get my work done by 4 pm." is the ___________________________.

"We will go for a walk." is the ________________________________.

There are four possibilities:

1. I get my work done by 4 pm; We go for a walk.
2. I get my work done by 4 pm; We don't go for a walk.
3. I don't get my work done by 4 pm; We go for a walk.
4. I don't get my work done by 4 pm; We don't go for a walk.

When is the promise broken? when the antecedent is true but the consequent is false $\boxed{T \rightarrow F}$

Truth Table for the Conditional:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

read if p then q

or

p implies q

vacuously true

Important note:

An implication $p \rightarrow q$ is false only when p is true and q is false.

Hidden Conditional Statements:

Examples: Re-write using the if…then…connective.

1. We will go for a walk if I get my work done by 4 pm.

2. Every picture tells a story.

If it's a picture, then it tells a story.

3. No guinea pigs are scholars.

If it's a guinea pig, then it's not a scholar.

4. It is difficult to study when you are distracted.

If you are distracted, then it's difficult to study.

Coming up:

1) Final Exam : Tuesday, Dec 10th 12pm – 1:50pm
here

- cumulative
- mostly multiple choice questions (around 40)
- best 40 out of 45
- open notes / calculator
- will replace a lowest test score

2) Turn in any late assignments by Tuesday 12 pm (written Hw / online hw)

Example: Translate the following statements into symbols given that

p : I drive to work.

q : The dog eats my shoes.

r : Milk costs \$3.50 per gallon.

① $\sim p \rightarrow r$

② $p \vee (\sim q \rightarrow r)$

1. If I do not drive to work, then milk costs \$3.50 per gallon.
2. I drive to work or if the dog does not eat my shoes then milk costs \$3.50 per gallon.

- $p \vee q$ is only false when both p and q are False.
- $p \wedge q$ is only true when both are true.
- $p \rightarrow q$ is only false when p is true and q is false.

Translate into words: $(\sim r \vee \sim p) \rightarrow \sim q$ using the simple statements above.

If milk doesn't cost 3.50 or I don't drive to work, then dog didn't eat my shoes

Examples: Determine the truth value of the conditional.

1. $F \rightarrow (4 \neq 7)$ $F \rightarrow T$: True

2. $(4 = 11 - 7) \rightarrow (8 < 0)$
   $T \rightarrow F$: False

3. Let the statement p be false, q be true, and r be false. What is the truth value of $(\sim r \vee p) \rightarrow q$?
   $T \vee F$
   $T \rightarrow T$: True

Example: Explain why in the statement $p \rightarrow [(\sim r \vee q) \wedge (\sim q \vee p)]$ the statement is true if all we know is that p is false?

Construct the truth table for the statement: $(p \wedge q) \rightarrow (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Tautology: A statement that is always true no matter the truth values of its components is called a tautology.

Example: Construct the truth table for the statement: $\left[(p \wedge r) \wedge (p \wedge q)\right] \rightarrow p$

p	q	r	$p \wedge r$	$p \wedge q$	$(p\wedge r)\wedge(p\wedge q)$	$(p\wedge r)\wedge(p\wedge q) \rightarrow p$
T	T	T				T
T	T	F				T
T	F	T				T
F	T	T				T
T	F	F				T
F	T	F				T
F	F	T				T
F	F	F				

next class

Equivalent Statements: Two statements are equivalent if they have the same truth value for every possible situation.

Notation for equivalent statements: $\equiv$

Example: Show that $p \rightarrow q \equiv \sim p \vee q$

p	q	$\sim p$	$p \rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

since these two columns have same truth values for every possible situation we say $p \rightarrow q$ is equivalent to $\sim p \vee q$.

Example: Show that $\sim(p \rightarrow q) \equiv p \wedge \sim q$

p	q	$\sim q$	$p \rightarrow q$	$\sim(p \rightarrow q)$	$p \wedge \sim q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

$\sim$ (If I get my work done by 4pm, then we will go for a walk)

I get my work done by 4pm and I don't go for a walk.

Fill in the following truth table:

T → F   T → F   T → F

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Definitions: For the conditional $p \to q$

Converse: $q \to p$

Inverse: $\sim p \to \sim q$

Contrapositive: $\sim q \to \sim p$

Equivalences:

$p \to q \equiv \sim q \to \sim p$

conditional $\equiv$ contrapositive

$q \to p \equiv \sim p \to \sim q$

converse $\equiv$ inverse

Example: Write the following for the conditional statement: If $\underbrace{\text{you build it}}_{p}$, $\underbrace{\text{he will come}}_{q}$.

Converse: If he will come, then you build it. $(q \to p)$

Inverse: If don't build it, then he we won't come $(\sim p \to \sim q)$

Contrapositive: If he doesn't come, then he won't build it. $(\sim q \to \sim p)$

Example: If $\underbrace{n^2 \text{ is odd}}_{p}$, then $\underbrace{n \text{ is odd}}_{q}$. Write the contrapositive of this conditional.

If n is even, then n^2 is even.

Alternative ways to say "If p, then q.

Example: If I live in Chicago, then I live in Illinois.

1. Living in Chicago is sufficient for living in Illinois.
2. Living in Illinois is necessary for living in Chicago.
3. I live in Chicago only if I live in Illinois.
4. All people who live in Chicago live in Illinois.
5. I live in Chicago implies I live in Illinois.
6. I live in Illinois if I live in Chicago.
7. I live in Illinois whenever I live in Chicago.

Biconditional: p if and only if q (p is necessary and sufficient for q)

Symbol: $p \Leftrightarrow q$

Meaning: $(p \to q) \wedge (q \to p)$

biconditional is true when both have same truth values

What is the truth table for the biconditional?

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

True or False:

1. $\overset{T}{3+1 \neq 6}$ if and only if $\overset{F}{8 \neq 8}$. : F

2. $6 \times 2 = 14$ if and only if $9 + 7 \neq 16$.

3. $12 \times 1 \neq 13$ if and only if $9 \neq 7$.

Construct the truth table for the statement:

$$(p \wedge \sim q) \leftrightarrow [(\sim p \vee q) \rightarrow p]$$

				C5		C7	C5 ⟺ C7
p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee q$	$(\sim p \vee q) \rightarrow p$	$(p \wedge \sim q) \leftrightarrow [(\sim p \vee q) \rightarrow p]$
T	T	F	F	F	T	T	F
T	F	F	T	T	F	T	T
F	T	T	F	F	T	F	T
F	F	T	T	F	T	F	T